**Murang'a University College**

*(A Constituent College of Jomo Kenyatta University of Agriculture and Technology)*

*University Examination*

**School of Pure & Applied Sciences**

**School of Computing & Information Technology**

**Supplementary/Special Examination for the degree of;**

**Bachelor of Science in Mathematics & Computer Science - Year TWO**

**Bachelor of Science in Information Technology - Year THREE**

**SMA 2230 PROBABILITY & STATISTICS II.**

Date: June/July 2016 2 Hours

INSTRUCTIONS: Attempt Question **One** and any other **Two** Questions.

## Question One (30 Marks)

(a) A random variable $X$ has the following distribution:

| $X$ | -1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $P(X=x)$ | $\frac{1}{3}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{3}$ |

Find the variance of $X$. **(5 Marks)**

(b) If the sum of mean and variance of a binomial distribution is 4.8 for five trials, find the distribution. **(5 Marks)**

(c) In an attempt to assess the physical condition of joggers, a sample of $n_1 = 25$ joggers was selected and their maximum volume of oxygen uptake ($VO_2$) was measured with the following results:

$$\overline{x}_1 = 47.5\text{mL/kg} \qquad s_1 = 4.8\text{mL/kg}.$$

Results for a sample of $n_2 = 26$ nonjoggers were

$$\overline{x}_2 = 37.5\text{mL/kg} \qquad s_2 = 5.1\text{mL/kg}.$$

Compute the two-sample test statistic. **(5 Marks)**

(d) The density of a random variable is given by

$$f(x) = \begin{cases} \frac{1}{2}x & 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find the expected value of $X$. **(5 Marks)**

(e) If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2,000 individuals, exactly 3 individuals will suffer a bad reaction. **(5 Marks)**

(f) Prove that $\text{Var}[ag(X) + b] = a^2\text{Var}[g(X)]$. **(5 Marks)**

## Question Two (20 Marks)

a) (i) A fair coin is tossed 6 times; call heads a success. What is the probability that exactly 2 heads occur? **(5 Marks)**

(ii) Suppose we are looking at the same experiment as the one in (a) (i) above. What is the probability that at least 4 heads occur? **(5 Marks)**

(b) If $X \sim B(20, 0.4)$

(i) find $P(6 \leq X \leq 10)$. **(5 Marks)**

(ii) Also find approximations to this probability using Poisson distribution. **(5 Marks)**

## Question Three (20 Marks)

(a) The probability that a marksman hits a target is $p = \frac{1}{4}$. She fires 100 times. Calculate

(i) the expected number $\mu$ of times she will hit the target and **(5 Marks)**

(ii) the standard deviation $\sigma$. **(5 Marks)**

(b) Let $X$ be a binomial random variable with $n$ trials and probability $p$ of success. Compute

(i) the first moment $M'(t)$ **(4 Marks)**

(ii) the second moment $M''(t)$. **(2 Marks)**

(iii) Use (b) (i) and (ii) above to compute $\text{Var}(X)$. **(4 Marks)**

**Question Four (20 Marks)**

(a) Suppose $X_1$ and $X_2$ are random variables representing serum-creatinine levels for white and black individuals with end-stage renal disease.

(i) Represent the sum, difference, and average of the random variables $X_1$ and $X_2$ as linear combinations of the random variables $X_1, X_2$. **(5 Marks)**

(ii) If we take the expected values of the white and black individuals as 1.3 and 1.5, respectively, what is the variance of the average serum-creatinine level over a single white and a single black individual? **(5 Marks)**

(b) Prove that a binomially distributed random variable with parameters $n$ and $p$ has mean $np$ and variance $npq$. **(10 Marks)**

**Question Five (20 Marks)**

(a) Suppose that a game is to be played with a single die assumed fair. In this game a player wins ksh 2,000 if a 2 turns up; ksh 4,000 if a 4 turns up; loses ksh 3,000 if a 6 turns up; while the player neither wins nor loses if any other face turns up.

(i) Find the expected sum of money to be won. **(5 Marks)**

(ii) Find the variance for the game. **(5 Marks)**

(b) The extent to which an infant's health is affected by parental smoking is an important public health concern. The following data are the urinary concentrations of cotinine (a metabolite of nicotine); measurements were taken both from a sample of infants who had

been exposed to household smoke and from a sample of unexposed infants.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Unexposed ($n_1 = 7$) | 8 | 11 | 12 | 14 | 20 | 43 | 111 | |
| Exposed ($n_2 = 8$) | 35 | 56 | 83 | 92 | 128 | 150 | 176 | 208 |

Compute the two-sample test statistics. **(10 Marks)**