Murang'a University College

(A Constituent College of Jomo Kenyatta University of Agriculture and Technology)

University Examination 2015/2016

School of Pure and Applied Science

Supplementary Examination for the Degree of Bachelor of Science in Mathematics and Computer Science - Year III

SMA 2321: NUMERICAL ANALYSIS - I

Date: June/July 2016 2 Hours

Instructions: Attempt Question **One** and any other **Two** Questions.

Question One (30 Marks)

a) Show that the local truncation error in Trapezoidal rule is given by
$R_T(x) = -\frac{h^3}{12}f''(\xi)$ (3 Marks)

b) Estimate the error when evaluating $f(x,y,z) = xy - \frac{2y}{z}$ when $x = 2.14$, $y = 4.108$ and $z = 8.215$ where each of x, y and z is assumed to be correctly rounded-off to the number of significant figures shown (5 Marks)

c) A polynomial of degree 3 is defined by the following table where one of the entry is suspected to be incorrect. By constructing the difference table, locate the incorrect entry hence compute the correct value

x	-2	-1.7	-1.4	-1.1	-0.8	-0.5	-0.2	0.1	0.4	0.7	1.0
f(x)	-15	-7.716	-2.448	1.128	3.336	4.05	4.944	4.992	4.968	5.196	6.0

(6 Marks)

d) Use the Lagrange interpolating polynomial to compute *f(4.2)* from the table below

x	-1	2	3	5
f(x)	13	1	1	3

(3 Marks)

e) Show that the Newton-Raphson iterative formula for determining the root of the equation $e^x - 3x^2 - 5 = 0$ is given by

$$X_{n+1} = \frac{(X_n - 1)e^{X_n} - 3X_n^2 + 5}{e^{X_n} - 6X_n}$$

(3 Marks)

f) Linearize the equation $y = ax^b$ hence give the associated normal equations for the least squares fit to the discrete data $(x_i,\ y_i)$, i=1, 2, ..., n (5 Marks)

g) Compute $f'(1.0)$ given the data in the table below

x	1	1.5	2	2.5
$f(x)$	-4.00	-2.7487	0.9289	7.7193

(5 Marks)

Question Two (20 Marks)

a) i) Given that $f(2.5) = 0.13793$ and $f(2.7) = 0.12063$, use linear interpolation to calculate the approximate value $f(2.55)$. (2 Marks)

ii) Given that $f(x) = \frac{1}{1+x^2}$, determine the bound on the truncation error for the method in part (i) above (4 Marks)

b) Determine the maximum step size that can be used in the tabulation of $f(x) = (1+x)^6$ in the interval [0, 1] so that the error in linear interpolation will be less than 5×10^{-3} (6 Marks)

c) i) Prove that $D = \frac{1}{h} ln(1 + \Delta)$
where D is the differential operator, h is the step size and Δ is the forward difference operator (4 Marks)

ii) Apply the Gregory-Newton forward formula to find a polynomial of the highest degree which takes the values in the table below

x	1	3	5
$f(x)$	15	23	40

(4 Marks)

Question Three (20 Marks)

a) Given the data in the table below, compute $f'''(0.2)$

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	4.0	3.608	3.264	3.016	2.912

(4 Marks)

b) Given the data in the table below

x	0.6	0.8	0.9	1.0	1.1	1.2	1.4
$f(x)$	0.707178	0.859892	0.925863	0.984007	1.033743	1.074575	1.127986

Use the central difference two-point formula and Richardson extrapolation to find $f'(1.0)$ (6 Marks)

c) Use the principle of least squares to fit an equation of the form $y = \frac{x}{a+bx}$ to the data in the table below

x	8	10	15	20	30	40
y	13	14	15.4	16.3	17.2	17.8

(10 Marks)

Question Four (20 Marks)

a) The Trapezoidal rule can be expressed in the form

$$\int_a^b f(x)dx = \frac{h}{2}(f_0 + 2(f_1 + f_2 + \ldots + f_{N-1}) + f_N) + R_t$$

where R_t is the Global error. Show that R_t is of $O(h^2)$ (4 Marks)

b) Determine the values of the constants a, b and c such that the formula

$$\int_0^1 f(x)dx = af\left(\frac{1}{3}\right) + bf\left(\frac{2}{3}\right) + cf(1)$$

is exact for the polynomial of as high degree as possible. (7 Marks)

c) Determine the truncation error for the method in part (b) above (4 Marks)

d) Use the Gauss-Legendre two point formula to evaluate the integral

$$I = \int_0^1 \frac{x}{1+x^2}dx$$

(5 Marks)

Question Five (20 Marks)

a) State the number of positive, negative and complex roots of the equation $x^3 + 4x + 5 = 0$ (3 Marks)

b) Show that the equation $5x^3 + x^2 + 7 = 0$ has root in the interval $[-2, -1]$ hence perform three iterations of the Regular-Falsi method to find the root correct to 4 decimal places. (4 Marks)

c) Use Newton-Raphson method to find the root of the equation $e^x + 2x - 3 = 0$ which is near $x = 1$ correct to 4 decimal places (5 Marks)

d) Derive the Gauss-Legendre 3-point formula $(n = 2)$ (8 Marks)