



MURANG'A UNIVERSITY COLLEGE

A constituent college of Jomo Kenyatta University of Agriculture and Technology

University Examination 2015/2016

**END OF SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF
SCIENCE IN MATHEMATICS & COMPUTER SCIENCE YEAR 3 SEMESTER 1**

SMA 2320: ANALYTICAL APPLIED MATHEMATICS I

DATE: DECEMBER 2015

TIME: 2 HOURS

Instructions: Answer question **One** and **Two** other questions

Question One (30 Marks)

- a) Find and classify the singular points of the differential equation:

$$x^2(x-2)^2 \frac{d^2y}{dx^2} + 2(x-2) \frac{dy}{dx} + (x+1)y = 0. \quad (4 \text{ Marks})$$

- b) Expand $1 + x - x^2 + x^3$ in a series of Legendre polynomials.

(5 Marks)

- c) Find the roots of the Indicial Equation of the differential equation:

$$4x \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + y = 0. \quad (6 \text{ Marks})$$

- d) Show that $\frac{d}{dx} [x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$.

(5 Marks)

- e) Show that $\int_{-1}^1 p_m(x) p_n(x) dx = 0$, for $m \neq n$, where $p_m(x)$ and $p_n(x)$ are Legendre polynomials.

(5 Marks)

- f) Find the general solution of $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3z$

(5 Marks)

Question Two (20 Marks)

- a) Find the Indicial Equation and the recurrence relation for the differential equation
- $$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0. \quad (10 \text{ Marks})$$
- b) Using the roots of the Indicial Equation in part (a) above, and the recurrence relation for the differential equation $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$, determine the complete solution for the differential equation. (10 Marks)

Question Three (20 Marks)

- a) Define Gamma function and hence show that:
- (i) $\Gamma(1) = 1$ (3 Marks)
- (ii) $\Gamma(n+1) = n\Gamma(n)$ (3 Marks)
- (iii) $\Gamma(n) = \int_0^\infty e^{-t} t^{n-1} dt$ (4 Marks)
- b) (i) Define Beta function and hence find the relation between Gamma and beta functions.
- (ii) Prove that $B(p, q) = \int_0^\infty \frac{y^{q-1}}{1+y^{p+q}} dy$. (10 Marks)

Question Four (20 Marks)

- a) Find the Indicial Equation and the recurrence relation for the differential equation
- $$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0. \quad (10 \text{ Marks})$$
- b) Using the roots of the Indicial Equation in part (a) above, and the recurrence relation for the differential equation $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0$, show that the complete solution of the differential equation may be written in the form $y = AJ_n(x) + BJ_{-n}(x)$, where A and B are arbitrary constants. (10 Marks)

Question Five (20 Marks)

a) Find the general solution of $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} + t \frac{\partial z}{\partial t} = xyt$.

(6 Marks)

b) Solve the equation $z \frac{\partial z}{\partial x} - z \frac{\partial z}{\partial y} = z^2 + (x + y)^2$.

(6 Marks)

c) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} - 6 \frac{\partial^2 z}{\partial y^2} = x + y$.

(8 Marks)