



**MURANG'A UNIVERSITY COLLEGE**

*A constituent college of Jomo Kenyatta University of Agriculture and Technology*

*University Examination 2015/2016*

**END OF SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF  
SCIENCE IN INMATHEMATICS & COMPUTER SCIENCE YEAR 3 SEMESTER 1**

**SMA 2305: COMPLEX ANALYSIS I**

**DATE: 7<sup>th</sup> DECEMBER 2015**

**TIME: 2 HOURS**

**Instructions:** Answer question **One** and **Two** other questions

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**Question One (30 Marks)**

- a) Given that  $z_1 = 1 - 3i$  and  $z_2 = -2 + 5i$ , determine  $\frac{z_1 z_2}{z_1 + z_2}$ . (4 Marks)
- b) Evaluate the limits:
- (i)  $\lim_{z \rightarrow 2} \frac{z^2 + 3}{iz}$  (2 Marks)
- (ii)  $\lim_{z \rightarrow 2i} \frac{z^2 + 4}{2z^2 + (3 - 4i)z - 6i}$  (3 Marks)
- c) Express the function  $w = \frac{z^2 + 1}{z}$  in the form  $w = u(x, y) + iv(x, y)$ . (5 Marks)
- d) (i) Find the derivative of  $f(z) = \frac{z^3 - 2z}{z - 1}$ . (3 Marks)
- (ii) Show that  $f(z) = \bar{z}$  is not analytic. (2 Marks)
- e) Evaluate  $\int z^2 dz$  along the curve C given by the parameterization  $x = t^2$  and  $y = t$  between  $t = 1$  and  $t = 2$ . (4 Marks)
- f) Evaluate  $\int \frac{e^z dz}{z^2 - 16}$  around the circle of radius 3 units centered at zero. (3 Marks)
- g) Evaluate  $\oint \frac{\cos \pi z^2 dz}{(z-1)(z-2)}$  around the circle given by  $|z| = 3$ . (4 Marks)

**Question Two (20 Marks)**

- a) Given that  $z = 1 + i$ , and  $f(z) = z^2 + 2z + 1$ , evaluate  $|f(z)|$  and  $\text{Arg}[f(z)]$ . (6 Marks)
- b) Use De Moivre's theorem to expand  $\cos 4\theta$  in terms of  $\cos \theta$ . (5 Marks)
- c) Find all the cube-roots of the complex number  $z = -1 + i$  and represent them in an Argand diagram. (9 Marks)

**Question Three (20 Marks)**

- a) Find a solution to the equation  $\cos z = i$ . (5 Marks)
- b) Evaluate  $\lim_{z \rightarrow e^{\frac{\pi i}{3}}} \frac{z(z - e^{\frac{\pi i}{3}})}{z^3 + 1}$ , giving your answer in the form  $a + ib$ . (5 Marks)
- c) Find  $u(x, y)$  and  $v(x, y)$  such that  $z \cosh z = u + iv$  where  $z = x + iy$  and  $x, y, u$  and  $v$  are real. (5 Marks)
- d) Locate and name all the singularities of the function  $\frac{(z+3i)^5}{(z^2-2z+5)^2}$ . (5 Marks)

**Question Four (20 Marks)**

- a) Prove that  $u = 3x^2y + 2x^2 - y^3 - 2y^2$  is harmonic and find a function  $v$  such that  $f(z) = u + iv$  is analytic. Express  $f(z)$  in terms of  $z$ . (8 Marks)
- b) State and prove Cauchy's integral theorem. (5 Marks)
- c) Evaluate  $\int \frac{1}{z-z_0} dz$  along a simple closed contour  $C$  having the origin as an interior point. (7 Marks)

**Question Five (20 Marks)**

a) Using Laurent's Theorem, prove that  $\oint f(z)dz = 2\pi iR$ , where  $f(z)$  is analytic inside and on a simple closed curve  $C$  except at one point  $z = a$  inside  $C$ ,  $R$  being the residue at  $z = a$ . (6 Marks)

b) Find the residues of:

(i)  $f(z) = \left(\frac{z+1}{z-1}\right)^2$  at  $z = 1$ . (3 Marks)

(ii)  $f(z) = z^{-2}\cot z$  at the origin. (4 Marks)

c) Using the Cauchy integral formulae, find the values of:

(i)  $\oint \frac{\sin^6 z}{z - \frac{\pi}{6}} dz$ , and (3 Marks)

(ii)  $\oint \frac{\sin^6 z}{(z - \frac{\pi}{6})^3} dz$  if  $C$  is the circle  $|z| = 1$  (4 Marks)