MURANG'A UNIVERSITY COLLEGE

(A Constituent college of Jomo Kenyatta University of Agriculture and Technology

SEM 1301: ENGINEERING MATHEMATICS V

INSTRUCTIONS

ANSWER QUESTION **ONE** AND **ANY TWO** QUESTIONS FROM THE OTHER QUESTIONS.

QUESTION ONE (30 MKS): COMPULSORY

a) Calculate the modulus of the sum of the vectors

$2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $3\mathbf{i} - 2\mathbf{j} + \mathbf{2k}$ and $5\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ (3mks)

b) Find the angle between the vectors $2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - \mathbf{k}$ (3mks)

c) Calculate the Curl of the vector $\mathbf{F} = 2xyz\mathbf{i} + 3x^2y\mathbf{j} + (xz^2 - y^2z)\mathbf{k}$ at

point $p(2,0,-3)$. (4mks)

d) Find the determinant of the following matrix: (3mks)

$$\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & -2 \\ -1 & 2 & 3 \end{bmatrix}$$

e) Find the area of a parallelogram having diagonals $\mathbf{A} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and

$\mathbf{B} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ (4mks)

f) Define the following terms: (2mks)

i. Interpolation

ii. Extrapolation

g) In the following problem, the values of a function f(x) are given. Find the interpolating polynomial that fits the data.

x	-1	1	4	7
f(x)	-2	0	63	342

Hence, find the approximation to f(x) at the point $x = 5.0$. (5mks)

h) Show that the vector field $\mathbf{A} = (6xy + z^3)\mathbf{i} + (3x^2 - Z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$ is irrotational. (3mks)

i) Evaluate $\int_v (2x + y)\,dv$ where v is the closed region bounded by the surface $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$ (3mks)

SECTION B (ATTEMPT ANY TWO QUESTIONS)

QUESTION TWO (20mks)

a) Find the unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$. (4mks)

b) Evaluate the following:

i. $\mathbf{k}.(\mathbf{i} + \mathbf{j})$

ii. $(\mathbf{i} - 2\mathbf{k}).(\mathbf{j} + 3\mathbf{k})$

iii. $(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}).(3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$

c) If $\mathbf{F} = (3x - 2y)\mathbf{i} + (y + 2z)\mathbf{j} - x^2\mathbf{k}$, evaluate $\int_c \mathbf{F}.d\mathbf{r}$ from (0,0,0) to (1,1,1), where c is a path consisting of the curve $x = t, y = t^2$ and $z = t^3$. (5mks)

d) If $\varphi = x^2yz + xz^2$ and $\emptyset = xy^2z - z^3$ evaluate $\nabla(\varphi + \emptyset)$at point $(-2, 0, -1)$ (4mks)

QUESTION THREE(20mks)

a) Find :

i. $\nabla\varphi$ if $\varphi(x, y, z) = 2x^2 - 3xy + 9z - 2$ at the point $(1,0,0)$ (3mks)

ii. The directional derivative at the point $(1,0,0)$ in the direction $(2, -2,1)$ (3mks)

b) Construct the Newton divided difference interpolation for the data in the table below:

x	0.5	1.5	3.0	5.0	6.5	8.0
f(x)	1.625	5.875	31.0	131.0	282.125	521.0

Hence , find the interpolating polynomial and use it to approximate the value of $f(7)$. (6mks)

c) Solve the following system of linear equations using Cramers' method. (5mks)

$$2x + y + z = 1$$
$$3x + 2y + z = 2$$
$$2x + y + 2z = -1$$

d) Given that $\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{B} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{C} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$,

Find $\mathbf{B}.(\mathbf{A} \times \mathbf{C})$ (3mks)

QUESTION FOUR(20mks)

a) Show that:

i. $\mathbf{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative field (3mks)

ii. Find the scalar potential. (4mks)

iii. Find the work done on a particle in moving it from position $(0,1,2)$ and $(5,6,8)$ (3mks)

b) Find the divergence of the vector field $\mathbf{B}(x, y, z) = xz^2\mathbf{i} - 2x^2yz^4\mathbf{j} + 2yz^4\mathbf{k}$ at $p(2, -2,1)$ (3mks)

c) Determine the values of a so that $\mathbf{A} = a\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2a\mathbf{i} + a\mathbf{j} - 4\mathbf{k}$ are perpendicular. (3mks)

d) If $\mathbf{A} = x^2yz\mathbf{i} - 2xz^3\mathbf{j} + xz^2\mathbf{k}$ and $\mathbf{B} = 2z\mathbf{i} + y\mathbf{j} - x^2\mathbf{k}$, find:

$\frac{\partial^2}{\partial x\,\partial y}(\mathbf{A} \times \mathbf{B})$ at (1,-1,3) (4mks)

QUESTION FIVE (20mks)

a) Evaluate $\int_V \mathbf{F}.d\mathbf{V}$ where $\mathbf{F} = 2\mathbf{i} + 2z\mathbf{j} + y\mathbf{k}$ and V is the region bounded by the planes $z = 0, z = 4$ and the surface $x^2 + y^2 = 9$. (Hint: Use the cylindrical coordinate system to evaluate the integral above) (10mks)

b) If $F = (2xy\mathbf{i} + (x^2 - z^2)\mathbf{j} - 3xz^2\mathbf{k},$ evaluate the line integral $\int_{AB} \mathbf{F}.d\mathbf{r}$ between A(0,0,0) and B(2,1,3) along the straight line from A(0,0,0) to $A_1(0,1,0)$, then to $A_2(2,1,0)$ and finally to B(2,1,3). (10mks)