MURANG'A UNIVERSITY COLLEGE
End of Semester Special/Supplementary Examination

SEB 1121 Engineering Mathematics 2 for Diploma in Civil Engineering*Time: 2h*
2015/2016 Academic Year

Date: 28th June, 2016 **time: 2 hours**

Answer Question ONE and ANY other two questions

QUESTION ONE (30 MARKS)

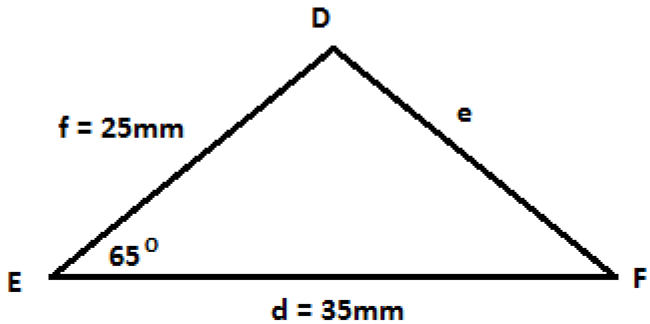
(a) From first principles show that sinθ in the third quadrant of the circle has a negative magnitude (2marks)

(b) Find the values of θ that satisfy the equation

(a) **Sin θ = - ½** ($0^o \leq \theta \geq 360^o$) (3 Marks)

(c) If ***Sin* θ *= 2/3*** find without using mathematical tables nor a calculator the values of Cos θ using the appropriate trigonometric identities (3 marks)

(d) Find the perimeter of triangle **DEF**. (5marks)



(e) Simplify j^3 (2 marks)

(f) Mark on an Argand diagram the radius vectors corresponding to
(a) ***- 3+2j*** (3 marks)
(b) ***Cos 180^0 + j Sin 180^0*** (3Marks)

(g) Determine the argument and modulus of ***2 + j 4*** (4 marks)

(h) Express ***3 + j 5*** into polar form (3 marks)

(i) Convert 120^0 to radians (2 Marks)

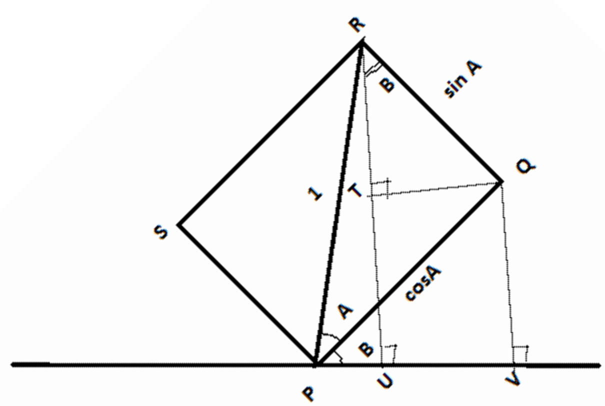
QUESTION TWO (20 MARKS)

(a) Find the value of Cos (45 – 30) without using mathematical tables nor a calculator (5marks)

(b) On a graph paper plot the graphs of ***y = Sin x*** and ***y = Cos 1.5x*** on the same axes for x values from 0^o to 90^o (using intervals of 10^o i.e. 10, 20, 30…). From the graph find the roots of the equation ***Sin x = Cos 1.5x*** (5Marks)

(c) Solve the equation **$2\,Sin^2\theta = Sin\,\theta$** for θ values from 0^o to 360^o inclusive (5 Marks)

(d) Study the architectural design of a rectangular building set on one of its vertices P. given that **RU = RT + TU** proof that ***Sin (A+B) = Sin A Cos B + Sin B Cos A*** (5 Marks)



QUESTION THREE (20 MARKS)

(a) Simplify $\mathbf{(1+i)^2}$, $\mathbf{(1+i)^3}$, $\mathbf{(1+i)^4}$

(b) Draw the Argand diagram radius vectors corresponding to $\mathbf{(1+i)}$, $\mathbf{(1+i)^2}$, $\mathbf{(1+i)^3}$, $\mathbf{(1+i)^4}$

(c) Find the principal values of the arguments of these complex numbers. (7 Marks)

(d) Express $\mathbf{5(Cos225^0 + j\ Sin225^0)}$ in the form $\mathbf{a + \mathit{j}\, b}$ (5 Marks)

(e) Express, $z = 10L37^0\ 1S'$ into exponential form (3 Marks)

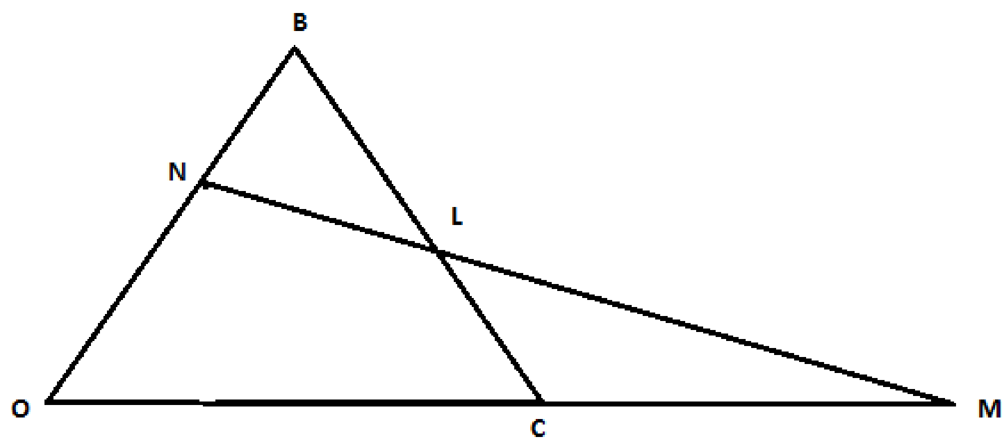
(f) Express, $z = e^{(1+j\,\pi/2)}$ into $a + jb$ form. (3 Marks)

QUESTION FOUR (20 MARKS)

(a) Express (-5, -12) into polar form (3 Marks)

(b) Express 4, 32^0 into Cartesian coordinates (4 Marks)

(b) Study the figure below. Given that vector ***ON*** = ¾ ***OB*** and vector ***BL*** = $\frac{2}{3}$***BC,*** Express vector ***NL*** in terms of ***b*** and ***c*** the position vectors of points B and C. (5 Marks)



(c) From first principles show how the CAST rule is obeyed

(1) All trigonometric ratios are positive in quadrant 1 (3marks)

(2) Only cosine is positive in quadrant 3 (3 Marks

(d) State any two applications of trigonometry in civil and building engineering

FORMULA SHEET

Trigonometrical identities

(a) $\sin^2\theta + \cos^2\theta = 1$; $\sec^2\theta = 1 + \tan^2\theta$; $\text{cosec}^2\theta = 1 + \cot^2\theta$

(b) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

(c) Let $A = B = \theta$ $\therefore$ $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

(d) Let $\theta = \frac{\phi}{2}$ $\therefore$ $\sin\phi = 2\sin\frac{\phi}{2}\cos\frac{\phi}{2}$

$$\cos\phi = \cos^2\frac{\phi}{2} - \sin^2\frac{\varphi}{2} = 1 - 2\sin^2\frac{\varphi}{2} - 2\cos^2\frac{\varphi}{2} - 1$$

$$\tan\phi = \frac{2\tan\frac{\phi}{2}}{1 - 2\tan^2\frac{\phi}{2}}$$

(e) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

(f) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

(g) Negative angles: $\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(h) Angles having the same trigonometrical ratios:

(i) Same sine: θ and $(180^\circ - \theta)$

(ii) Same cosine: θ and $(360^\circ - \theta)$, i.e. $(-\theta)$

(iii) Same tangent: θ and $(180^\circ + \theta)$

(i) $a\sin\theta + b\cos\theta = A\sin(\theta+\alpha)$

$a\sin\theta - b\cos\theta = A\sin(\theta-\alpha)$

$a\cos\theta + b\sin\theta = A\cos(\theta-\alpha)$

$a\cos\theta - b\sin\theta = A\cos(\theta+\alpha)$

where $\begin{cases} A = \sqrt{a^2+b^2} \\ \alpha = \tan^{-1}\frac{b}{a} \quad (0^\circ < \alpha < 90^\circ) \end{cases}$

sine of angle θ as $\frac{\text{opposite}}{\text{hypotenuse}} =$

cosine of angle θ as $\frac{\text{adjacent}}{\text{hypotenuse}}$

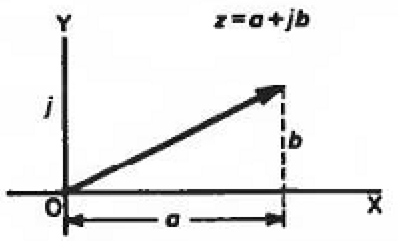
tangent of angle θ as $\frac{\text{opposite}}{\text{adjacent}} =$

Powers of j

$$j=\sqrt{-1},\quad j^2=-1,\quad j^3=-j,\quad j^4=1.$$

A factor j turns a vector through 90° in the positive direction.

Complex numbers



a =real part
b =imaginary part

Conjugate complex numbers $(a+jb)$ and $(a-jb)$
The product of two conjugate numbers is always real:

$$(a+jb)(a-jb)=a^2+b^2$$

Equal complex numbers

If $a+jb=c+jd$, then $a=c$ and $b=d$.

Polar form of a complex number

$$\begin{aligned} z &= a+jb \\ &= r(\cos\theta + j\sin\theta) \\ &= r\underline{|\theta} \end{aligned}$$

$$r=\sqrt{a^2+b^2};\quad \theta=\tan^{-1}\left\{\frac{b}{a}\right\}$$

also $\quad a=r\cos\theta;\ b=r\sin\theta$

$r=$ the modulus of z written 'mod z' or $|z|$

$\theta=$ the argument of z, written 'arg z'

Exponential form of a complex number

$$\left.\begin{aligned} z = r(\cos\theta + j\sin\theta) &= re^{j\theta} \\ \text{and} \qquad r(\cos\theta - j\sin\theta) &= re^{-j\theta} \end{aligned}\right\}\ \theta \text{ in radians}$$

Logarithm of a complex number

$$z=re^{j\theta} \qquad \therefore\ \ln z=\ln r+j\theta$$

$$\text{or if}\quad z=re^{-j\theta} \qquad \therefore\ \ln z=\ln r-j\theta$$

A *scalar* quantity has magnitude only; a *vector* quantity has both magnitude and direction.

The axes of reference, OX, OY, OZ, are chosen so that they form a right-handed set. The symbols **i**, **j**, **k** denote *unit vectors* in the directions OX, OY, OZ, respectively.

If $\overline{OP} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, then $|\overline{OP}| = r = \sqrt{a^2 + b^2 + c^2}$

The *direction cosines* $[l, m, n]$ are the cosines of the angles between the vector and the axes OX, OY, OZ respectively.

For any vector: $l = \frac{a}{r}$, $m = \frac{b}{r}$, $n = \frac{c}{r}$; and $l^2 + m^2 + n^2 = 1$

Scalar product ('dot product')

$\mathbf{a}.\mathbf{b} = ab\cos\theta$ where θ is the angle between **a** and **b**.
If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$
then $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$

Vector product ('cross product')
$\mathbf{a} \times \mathbf{b} = (ab\sin\theta)$ in direction perpendicular to **a** and **b**, so that **a**, **b** and $(\mathbf{a} \times \mathbf{b})$ form a right-handed set.

Also $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$