MURANG'A UNIVERSITY COLLEGE

(A Constituent College of Jomo Kenyatta University of Agriculture and Technology)

University Examination 2014/2015

YEAR II SEMESTER I SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY

ICS 2211: NUMERICAL LINEAR ALGEBRA

DATE: 7th August 2015 **TIME: 2 HOURS**

Instructions: Attempt question **One** and **Two** other questions

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Question One (30 Marks)

a) Given the matrices

$$A = \begin{pmatrix} 3 & 1 \\ -2 & 2 \\ 4 & 0 \end{pmatrix} \quad and \quad B = \begin{pmatrix} 2 & 6 & 7 \\ 5 & 8 & 3 \end{pmatrix}$$

Show that $A \cdot B$ is singular (4 marks)

b) Use iterative method to find the inverse of the matrix

$$A = \begin{pmatrix} 4 & 5 \\ 1 & -2 \end{pmatrix}$$

Given that its approximate inverse is $\begin{pmatrix} 0.1 & 0.4 \\ 0.1 & -0.3 \end{pmatrix}$ (*perform two iterations*) (5 marks)

c) Find the spectral norm $(\|A\|_2)$ of the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$$ (7 marks)

d) Consider the linear system of equations given by

$$7x_1 + 2x_2 - x_3 = 6$$
$$4x_1 - 8x_2 + 3x_3 = 36$$
$$3x_1 + 4x_2 - 10x_3 = -42$$

The Jacobi iterative method for the system is given by $X^{(k+1)} = HX^{(k)} + C$

Determine the Jacobi iterative matrix (H) of the system. (5 Marks)

e) Given two non-singular matrices A and B, prove that

$(AB)^{-1} = B^{-1}A^{-1}$ (5 Marks)

f) Given that $(1 \quad 1)^t$ is an Eigen vector of the matrix $A = \begin{pmatrix} 2 & b \\ 3 & 1 \end{pmatrix}$

Find the values of the constant *b* (4 Marks)

Question Two (20 Marks)

a) Use Cramer's rule to find the solution to the linear system of equations

$$4x - 5y - z = 9$$
$$3x - 2y + 2z = -9$$
$$x + 3y + 4z = 2$$ (12 marks)

b) Apply the method of steepest descent to find the local minimum of the function

$f(x) = x_1^2 + x_2^2$ given $X^{(0)} = (2,2)$ (8 marks)

Question Three (20 Marks)

a) Given that $A = PDP^{-1}$ where A is a non-singular matrix and D is a diagonal matrix, prove that $A^n = PD^nP^{-1}$ for any integer *n*. (8 marks)

b) Use Simplex method to find the optimum solution to the linear programming problem

$$Maximize \quad z = 4x + 2y$$
$$Subject\ to \quad 3x + 4y \leq 6$$
$$2x + 3y \leq 4$$
$$x, y \geq 0$$ (12 marks)

Question Four (20 Marks)

a) Find the Eigen values and the corresponding Eigen vectors of the matrix

$$A = \begin{pmatrix} 4 & -1 & -3 \\ 3 & 2 & -3 \\ 5 & -1 & -4 \end{pmatrix}$$ (13 marks)

b) Use power method to find the dominant Eigen value of the matrix

$$A = \begin{pmatrix} 3 & 2 & 3 \\ 2 & 6 & 6 \\ 3 & 6 & 3 \end{pmatrix}$$

Use $X^{(0)} = (1 \quad 1 \quad 1)^t$ and perform two iterations (7 marks)

Question Five (20 Marks)

a) Given the matrix $A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$ find a matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix hence find A^5. (11 marks)

b) Use matrix inversion method to find the solution to the linear system of equations;

$$2x + 3y + z = 8$$
$$4x + 6y + 5z = 4$$
$$3x - 4y + 2z = -7$$ (9 marks)