



**MURANG'A UNIVERSITY OF TECHNOLOGY**  
**SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES**  
**DEPARTMENT OF MATHEMATICS AND ACTUARIAL**  
**SCIENCE**

**UNIVERSITY POSTGRADUATE EXAMINATION**

**2023/2024 ACADEMIC YEAR**

**FIRSTYEAR FIRST SEMESTER EXAMINATION FOR MASTER OF**  
**SCIENCE IN AGRICULTURAL ECONOMICS**

**GAE 605: MATHEMATICS FOR ECONOMISTS**

**DURATION: 3 HOURS**

**INSTRUCTIONS TO CANDIDATES:**

1. Answer any four questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

### QUESTION ONE (25 MARKS)

- a) Find  $\frac{dy}{dx}$  given the functions below
- (i)  $y = (x^2 + e^x) \cos 2x$  (3 marks)
  - (ii)  $y = \frac{\sin 3x}{4x^3 - 8}$  (3marks)
- b) Use logarithmic differentiation method to find  $y = \frac{x^3 e^{2x}}{\cos x}$  (3marks)
- c) Find and classify the stationary points of the function  $z = x^3 - 6xy - y^3$  (9 marks)
- d) A company has established through statistical demand analysis that the demand  $x$  of the product is related to the price of  $p$  of the product by  $x = 1000 - 4p$   
while the cost  $c$  of the product is related to the demand of the product by the equation  $c = 6000 + 50x$ .
- i) Determine the level of output for which the revenue is maximised (3marks)
  - ii) Determine the level of output for which the profit is maximised (4marks)

### QUESTION TWO (25 MARKS)

- a) Given the function  $z = x^3 y^2 + 2xy^3$   
find  $\frac{dz}{dx}$  along the curve  $y = x^3$  (5marks)
- b) Optimize the function  $f(x, y) = xy^2 - 3x^2$  subject to the constraint  $x + y^2 = 1$  using the method of Lagrange multiplier. (7marks)
- c) The Cobb-Douglas production function for a new product is given by
- $$f(x, y) = 16x^{0.25}y^{0.75}$$

Where  $x$  is the number of units of labour and  $y$  is the number of units of capital required to produce  $f(x, y)$  units of the product. Each unit of labour costs \$ 50 and each unit of capital costs \$ 100. If \$500,000 has been budgeted for the production of this product,

- i) How should the amount be allocated between labour and capital in order to maximize production? (10 marks)
- ii) What is the maximum number of units that can be produced? (1mark)
- iii) Find the marginal productivity of money (1 Mark)
- iv) Estimate the increase in production when an additional \$ 90,000 is added into the budget. (1 mark)

### QUESTION THREE (25 MARKS)

- a) Find
- i)  $\int (x^2 + 3) \cos 2x \, dx$  (3 marks)
  - ii)  $\int 2x e^{x^2} \, dx$  (3 marks)
- b) Evaluate  $\int_0^2 \frac{x}{3x^2+5} \, dx$  (4marks)

- c) A man earns \$1,000 per month and spend his income on three products  $x, y$  and  $z$ . the cost of purchasing the three products are \$10, \$25 and \$ 40 respectively. The utility function for the three products is given by  $u = y^2 + 4x^2z$ .

Determine the quantity of each product that he should buy to maximise his utility (10marks)

- d) A company found that as a result of a new product recently introduce into the market, the sales has been increasing at a constant rate of 5% p.a. The sales in the year of introduction was 100 units. Construct the growth function hence estimate the sales in the 8<sup>th</sup> year.  
(5 marks)

#### QUESTION FOUR (25 MARKS)

- a) In a survey of 400 people, it was found that 200 like reading books, 180 liked watching movies and 160 liked listening to music. In addition, 80 liked both reading books and watching movies.70 liked both watching movie and listening to music, 60 liked both reading books and listening to music and 10 liked all the three activities.
- i) Represent this information in a Venn diagraph (2marks)
  - ii) Determine the number of person who liked...
    - I. Reading books only (1mark)
    - II. Exactly two activities (1mark)
    - III. Exactly one activity (1mark)
    - IV. None of the three activities (1 mark)
- b) A company plans to launch a new product in market. The research department estimates that the demand function for the product will be given by  $x = 5000 - 500p$ , where  $x$  is the demand and  $p$  is the price per unit. The production department on the other hand estimates that the fixed cost of production of the new product is \$ 1,000 while the variable cost is \$2 per unit. Find
- i) The cost function (1mark)
  - ii) Revenue function (3marks)
  - iii) Profit function (2mark)
  - iv) The break- even points (4marks)
- c) From past experience, a book store has determined that 8,000 books are sold on average in each quarter. It costs the bookstore kshs 30 to store book for one quarter. Every time the bookstore receives a new shipment for books from the publisher, the cost of handling and inspection is kshs 800. Determine the number of shipments and the number of books that the bookstore should order in each shipment in order to minimize the total costs. (9marks)

**QUESTION FIVE (25 MARKS)**

a) Given that the matrix A given below is singular, find the possible values of  $\lambda$

$$A = \begin{pmatrix} 2 & 4 & 0 \\ \lambda & 1 & 7 \\ 3 & -1 & \lambda \end{pmatrix} \quad (5\text{marks})$$

b) An economy consists of three independent sectors of Agriculture, construction and services. The inputs and outputs between the three sectors are represented by:

		Inputs (tons)			
Outputs (tons)	Agriculture	Construction	Services	Demand	
Agriculture	50	20	60	70	
Construction	40	30	20	70	
Services	60	40	50	50	

Find :

- i) The technical matrix (2marks)
  - ii) The Leontief matrix (2marks)
  - iii) The output level for each sector if the final demand for agriculture increases by 15 units, the demand for construction sector increases by 25 units and the demand for services sector decreases by 10 units. (6marks)
- c) A company produces two types of soft boards  $x$  type A and  $y$  type B ( where  $x$  and  $y$  are in thousands). The yearly profit function is given by

$$p(x, y) = -22x^2 + 22xy - 11y^2 + 110x - 44y - 23$$

Find:

- i) The marginal profit of type A boards when  $x = 2$  and  $y = 2$  (2 marks)
- ii) The marginal profit of type B boards when  $x = 2$  and  $y = 2$  (2marks)
- iii) Interpret the results in (i) and (ii) above (2marks)
- iv) Optimal production of type A and type B boards (3 marks)
- v) Optimal profit (1 mark)