MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY ORDINARY EXAMINATION

2023/2024 ACADEMIC YEAR

FOURTH YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

AMS 409 – STOCHASTIC PROCESSES II

DURATION: 2 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer question one and any other two questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a. Define the following terms
   i. Ergodic State (1 mark)
   ii. Transient State (1 mark)
   iii. Period of a state of Markov chains (1 mark)
   iv. Absorbing state (1 mark)

b. Find the generating function for the sequence
   {0, 0,0,7,7,7,7,…..} (2 marks)

c. Let x have a Bernoulli distribution with parameters p and q given by
   $p_r(X = k) = p_r = p^k q^{1-k}, q = 1 - p, k = 0,1.$
   Obtain the probability generating function of X and hence find its mean and variance. (6 marks)

d. Discuss two methods of finding probability mass functions of some independent random variables. (4 marks)

e. Derive birth-death process given that $Z(t)$ is the population size at time t, and t $p_n(t)$ is the probability that the population is of size n at time t. (6marks)

f. Explain three properties of a Poisson process (3 marks)

g. Suppose that λ is the rate of occurrence of a poisson process. Find the probability that there will be 3 arrivals in (0,4] and 3 arrivals in (2,5] given that $\lambda = 1$. (5 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a. Classify the states of the following transitional probability matrix (10 marks)

$$\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

b. Using Pure-Birth process difference – differential equation given by

$$\boldsymbol{p'_n(t) = -\lambda_n p_n(t) + \lambda_{n-1} p_{n-1}(t);\ n \geq 1 \text{ and } p'_0(t) = -\lambda_0 p_0(t)}$$

Obtain $\boldsymbol{p'_n(t)}$ for Yule – Ferry process when $\boldsymbol{\lambda_n} = n\lambda$ and hence find its mean and variance (10 marks)

QUESTION THREE (20 MARKS)

a. Consider a series of Bernoulli trials with probability of success p. Suppose that X denote the number of failures preceding the first success and preceding the second success. Find:

  i. The joint pdf of X and Y (1 mark)
  ii. Bivariate probability generating function of X and Y (3 marks)
  iii. Mean and variance of Y (4 marks)
  iv. Covariance of X and Y (2 marks)

b. The difference-differential equation for pure-birth process are

$$\boldsymbol{p'_n(t) = -\lambda_n p_n(t) + \lambda_{n-1} p_{n-1}(t);\ n \geq 1} \ \textbf{ and } \ \boldsymbol{p'_0(t) = -\lambda_0 p_0(t);\ n = 0}.$$

Obtain $\boldsymbol{p'_n(t)}$ for a non-stationary pure birth process (Poisson process) given that;

$$\boldsymbol{p_n(t)} = \begin{cases} 1 \ for\ n = 1 \\ 0, \ \ Otherwise \end{cases}$$

Hence obtain its mean and variance. (10 marks)

QUESTION FOUR (20 MARKS)

The difference-differential equation for simple-birth-death processes are;

$$\boldsymbol{p'_n(t) = -n(\lambda + \mu) p_n(t) + (n-1)\lambda p_{n-1}(t) + (n+1) p_{n+1}(t);\ n \geq 1} \ \textbf{and}$$

$\boldsymbol{p'_0(t) = \mu p_1(t);\ n = 0}$. Obtain $\boldsymbol{p'_n(t)}$ for a simple Birth-Death process with

$\boldsymbol{\lambda_n} = n\lambda \ and \ \mu_n = n\mu$ given that $\boldsymbol{p_n(t)} = \begin{cases} 1 \ for\ n = 1 \\ 0, \ \ Otherwise \end{cases}$ (20 Marks)