MURANG'A UNIVERSITY COLLEGE

A constituent college of Jomo Kenyatta University of Agriculture and Technology

University Examination 2015/2016

END OF SEMESTER SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF APPLIED STATISTICS WITH COMPUTING -YEAR 1 SEMESTER 2

AMS 2103: PROBABILITY AND DISTRIBUTION THERY 1

DATE: june 2016 **TIME: 2 HOURS**

Instructions: Attempt question **One** and **Two** other questions

QUESTION 1(30 MARKS)

a) State any two properties that probability density function of discrete random variables X should satisfy [2marks]

b) If X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} k(3-x)(1+x) \; for \;\; 0 \le x \le 3 \\ 0, \;\; elsewhere \end{cases}$$

and k is a positive constant. Determine

(i) value of the k [3marks]

(ii) $E(X)$ [3marks]

(iii) $Var(X)$ [3marks]

c) A random variable X has probability distribution $p(x)$ given below.

x	P(x)
0	0.2
1	0.3
2	A
3	b

Given that $E(x) = 1.4$, determine the probabilities a and b and hence the variance(X) [4marks]

d) The distribution function for a random variable X is

$$F(x) = \begin{cases} 1 - e^{-2x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

Find

(i) the density function, [2mark]

(ii) the probability that $x > 2$, [2mark]

(iii) the probability that $-3 < x \le 4$. [2marks]

e) A continuous random variable X has moment generating function $M(t) = e^{t^2+t}$. Determine the

(i) $E(X)$ and [2marks]

(ii) $Var(X)$ [2marks]

f) Of 50 building in an industrial park, 12 have electrical code violations . if 10 buildings are selected at random for inspection.

(i) what is the probability that exactly 3 of the 10 have a code violation [2marks]

(ii) Find the mean of this distribution [2marks]

QUESTION 2 (20 MARKS)

a) The data below shows the number of workers of a company who died due to a certain disease.

No of deaths	0	1	2	3	4	Total
Frequency	122	60	15	2	1	200

Fit a binomial distribution to the given data [7marks]

b) The distribution of the annual salaries in US dollars (US$) of mechanical engineers working in Kenya is a continous random variables X given by probability density function

$$\boldsymbol{f(x) = \begin{cases} b(9-x), & -3 \le x \le 3 \\ 0 & elsewhere \end{cases}}$$

Where $\boldsymbol{b}$ is a constant.

Determine

(i) The value of $\boldsymbol{b}$ [3marks]

(ii) Obtain the value of $\boldsymbol{P(x < 1)}$ [4marks]

(iii) Find the mean and the variance of $\boldsymbol{X}$. [6marks]

QUESTION 3 (20 MARKS)

a) In a game organized by Jeremiah the scores for the games is a random variable X which takes the set of values $\{0, 1, 2, 3, 4\}$ according to the probability distribution shown below.

$\boldsymbol{X}$	$\boldsymbol{P(x)}$
0	0.1
1	A
2	B
3	0.2
4	0.1

Given that the expectation of $\boldsymbol{X}$ is 1.8 determine the values of $\boldsymbol{A}$ and $\boldsymbol{B}$ hence determine the variance of $\boldsymbol{X}$. [7marks]

b) Let X be a continues random variable having pdf f(x) given by

$$f(x) = \begin{cases} 5x^4 \; for \;\; 0 \le x \le 1 \\ 0, \quad otherwise \end{cases}$$

(i) Determine the pdf of the continous random variable Y where $Y = X^3$ [3marks]

(ii) Determine $P\left(\frac{1}{2} < X < 1\right)$ [3marks]

(iii) Determine $P(\frac{1}{8} < y < 1)$ [3marks]

QUESTION 4 (20 MARKS)

a) Let X be a continous random variable having pdf $f(x)$ given by

$$f(x) = \begin{cases} \theta e^{-\theta x} \; for \; 0 \leq x \leq \infty \\ 0, \quad otherwise \end{cases}$$

Determine the mean and variance of X. [8marks]

b) Use the Poisson approximation to compute the probability $P(X \leq 2)$ where X is binomial with $n = 160$ and $p = 0.05$ [4marks]

c) If the random variable X has a geometric probability distribution for which the p.m.f is given by

$$\boldsymbol{f(x) = \begin{cases} p(1-p)^{x-1}, \; x = 1, 2 \ldots . \\ 0, \quad otherwise \end{cases}}$$

Obtain the mean of the random variable X [8marks]

QUESTION 5 (20 MARKS)

a) A random variable X has a gamma distribution with parameter $\alpha > 0$ and $\beta > 0$ whose pdf is given by

$$f(x) = \begin{cases} \dfrac{X^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^{\alpha}} \quad for \; x > 0, \alpha > 0 \\ 0, otherwise \end{cases}$$

Obtain the mean and variance of the random variable X. [10marks]

b) Let X be a discrete random variable having pmf $f(x)$ given by

$$P(x) = \begin{cases} \dfrac{1}{6}\left(\dfrac{5}{6}\right)^{x}, \quad x = 0,1,2 \ldots . \\ 0, otherwise \end{cases}$$

Determine the mean and variance of random variable X using moment generating function. [10marks]