Murang'a University College

(A Constituent College of Jomo Kenyatta University of Agriculture and Technology)

University Examination 2015/2016

School of Pure and Applied Science

Supplementary Examination for the Degree of Bachelor of Science in Applied Statistics with Programming - Year I

AMM 2102: FOUNDATION MATHEMATICS II

Date: 2016 2 Hours

Instructions: Attempt Question **One** and any other **Two** Questions.

Question One (30 Marks)

a) Find the value of λ for which the matrix

$$A = \begin{pmatrix} 3 & -1 & 5 \\ 7 & 6 & \lambda \\ 1 & 9 & -4 \end{pmatrix}$$ is singular (4 Marks)

b) Find the angle between the vectors $\vec{A} = -\tilde{i} + 4\tilde{j} - 5\tilde{k}$ and $\vec{B} = 4\tilde{i} - \tilde{j} + 7\tilde{k}$ (4 Marks)

c) Find a unit vector perpendicular to the vectors in part (b) above (5 Marks)

d) Use Gaussian elimination method to find the solution to the linear system of equations

$x + 2y + z = 8$

$2x + 3y + 4z = 20$

$4x + 2z = 10$ (5 Marks)

e) Find the value of λ for which the vectors $\vec{A} = 2\tilde{i} + 3\tilde{j} - \tilde{k}$, $\vec{B} = \lambda\tilde{i} + 2\tilde{j} + 2\tilde{k}$ and $\vec{C} = 3\tilde{i} + 4\tilde{j} + \tilde{k}$ are co-planar (4 Marks)

f) Given any two non-singular matrices A and B, prove that $(AB)^{-1} = B^{-1}A^{-1}$. (4 Marks)

g) Given the matrices

$$A = \begin{pmatrix} 5 & -1 & 2 \\ 3 & 1 & 4 \\ 1 & 4 & 1 \end{pmatrix} \quad \textit{and} \quad B = \begin{pmatrix} -15 & 9 & -6 \\ 1 & 3 & -14 \\ 11 & -21 & 8 \end{pmatrix}$$

Find the matrix product AB hence find A^{-1} (4 Marks)

Question Two (20 Marks)

a) Determine whether the vectors $\vec{A} = \tilde{i} + 7\tilde{j} + \tilde{k}$, $\vec{B} = 3\tilde{i} - 5\tilde{j} + 2\tilde{k}$ and $\vec{C} = 2\tilde{i} - 2\tilde{j} - \tilde{k}$ are linearly dependent or independent (4 Marks)

b) Given the vectors $\vec{A} = 4\tilde{i} + \tilde{j} - 2\tilde{k}$ and $\vec{C} = \tilde{i} - 6\tilde{j} - \tilde{k}$. Find another vector $\vec{B}$ such that $\vec{A} \times \vec{B} = \vec{C}$ and $\vec{A} \cdot \vec{B} = 5$ (7 Marks)

c) A particle moves in space so that at any time t, its position is given by $x = t^3 - 2$, $y = 2t^2 + t$ and $z = 2t - t^2$. Find the components of its velocity and acceleration in the direction of the vector $\vec{A} = 2\tilde{i} - \tilde{j} + 5\tilde{k}$ when $t = 1$ (9 Marks)

Question Three (20 Marks)

a) Find the determinant of the matrix

$$A = \begin{pmatrix} 5 & -2 & 1 \\ 7 & 2 & 9 \\ -1 & 2 & -5 \end{pmatrix}$$ (2 Marks)

b) Given the matrices

$$A = \begin{pmatrix} -1 & 2 & 3 \\ 1 & 2 & 2 \\ 4 & -1 & -3 \end{pmatrix} \quad and \quad B = \begin{pmatrix} 4 & 3 & 1 \\ -11 & -9 & -3 \\ 9 & 7 & 3 \end{pmatrix}$$

Compute the matrix product AB hence find the solution to the linear system of equations

$4x + 3y + z = 1$

$-11x - 9y - 3z = -2$

$9x + 7y + 3z = 4$ (7 Marks)

c) Use matrix inversion method to find the solution to the linear system of equations

$4x_1 + 5x_2 + x_3 = 2$

$x_1 - 2x_2 - 3x_3 = 7$

$3x_1 - x_2 - 2x_3 = 1$ (11 Marks)

Question Four (20 Marks)

a) A particle under the action of a constant force $\vec{F} = 2\tilde{i} + 5\tilde{j} - 3\tilde{k}$ is displaced from the point $(2, -3, 5)$ to the point $(1, 4, -1)$. Determine the work done by this force (3 Marks)

b) A force $\vec{F} = 3\tilde{i} - 9\tilde{j} + 2\tilde{k}$ is acting at a point P whose position vector is $6\tilde{i} + \tilde{j} - 5\tilde{k}$. Find the magnitude of the moment of this force about the point $Q(3, 4, -5)$ (6 Marks)

c) Given that the vectors $\vec{A} = 6\tilde{i} + \lambda\tilde{j} + \tilde{k}$ *and* $\vec{B} = 2\lambda\tilde{i} + \lambda\tilde{j} - 3\tilde{k}$ are orthogonal, find the possible values of λ (5 Marks)

d) Use the method of systematic elimination to find the solution to the linear system of equations

$x + 2y + z = 8$

$2x + 3y + 4z = 20$

$4x + 3y + 2z = 16$ (6 Marks)

Question Five (20 Marks)

a) Given the matrices $A = \begin{pmatrix} 1 & -1 \\ 3 & 2 \\ -1 & 3 \end{pmatrix}$ *and* $B = \begin{pmatrix} 2 & -4 & 1 \\ 7 & -5 & 0 \end{pmatrix}$

Show that AB is a singular matrix (4 Marks)

b) Use the Gaussian elimination method to find the inverse of the matrix below

$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 4 \\ 3 & 1 & 6 \end{pmatrix}$ (7 Marks)

c) Use Cramer's rule to find the solution to the linear system of equations

$x + 2y + 3z = -6$

$-2x + y + 2z = 1$

$3x - y + z = 1$ (9 Marks)