Murang'a University College

(A Constituent College of Jomo Kenyatta University of Agriculture and Technology)

University Examination 2015/2016

School of Pure and Applied Science

Special/Supplementary Examination for the Degree of Bachelor of Science in Applied Statistics with Programming Year - I

AMM 2101: FOUNDATION OF MATHEMATICS - I

Date: 2016 — 2 Hours

Instructions: Attempt Question One and any other Two Questions.

Question One (30 Marks)

a) In how many ways can 12 persons arrange to sit at a round table (2 Marks)

b) A committee of 8 persons is to be selected from 11 men and 7 women. In how many ways can this be done if the committee is to have at least 5 women? (4 Marks)

c) Show that 0.474747... is a rational number (3 Marks)

d) Show that $(p \vee q) \wedge ((\neg p) \wedge (\neg q))$ is a contradiction (3 Marks)

e) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 - 1$ is not injective (5 Marks)

f) Use the principle of mathematical induction to prove that
$1 + 3 + 5 + \ldots + (2n - 1) = n^2$ (5 Marks)

g) Construct the complete truth table for $(p \vee q) \leftrightarrow r$ (4 Marks)

h) Find the principle value of $(2i)^i$ (4 Marks)

Question Two (20 Marks)

a) State the two De-Morgan's law hence prove that $(p \vee q) \vee ((\neg p) \wedge (\neg q))$ is a tautology (4 Marks)

b) Use the principle of mathematical induction to prove that $23^n - 1$ is divisible by 11
$\forall \quad n \in \mathbb{N}$ (7 Marks)

c) Define a function $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = 4x - 3$.
Show that $f(x)$ is bijective hence find its inverse (9 Marks)

Question Three (20 Marks)

a) Given that $cos(\theta) = \frac{4}{5}$ and $sin(\alpha) = \frac{7}{25}$ where α and θ are acute angles, find the value of $cos(\theta - \alpha)$ without finding the values of $\alpha \quad \& \quad \theta$ (5 Marks)

b) Solve the equation $3cos(2x) - 9sin(x) + 5 = 0$ for $0 \leq \theta \leq 2\pi$ (6 Marks)

c) Express $5sin(\theta) - 12cos(\theta)$ into the form $Rsin(\theta + \psi)$ where R and ψ are constants hence solve the equation $5sin(\theta) - 12cos(\theta) = 7$ for $0 \leq \theta \leq 2\pi$ (9 Marks)

Question Four (20 Marks)

a) Prove that $(n+2)! + (n+1)! + n! = (n+2)^2 n!$ (5 Marks)

b) Find the value of n that satisfy the equation ${}^nC_3 =^n P_2$ (5 Marks)

c) Prove that ${}^nC_r =^n C_{n-r}$ where $r, n \in \mathbb{N} \quad and \quad r \leq n$ (4 Marks)

d) How many odd numbers greater than 40000 can be made with the digits 1, 3, 4, 7 and 9 given that each digit is to be used only once in each number (6 Marks)

Question Five (20 Marks)

a) Given that $S(0) = 1$, show that $\forall \quad b \in \mathbb{N}$, $S(b) = b + 1$ where $S(x)$ denote the successor of x $\forall \quad x \in \mathbb{N}$ (3 Marks)

b) There are 79 students who take Bachelor of Science programme. 41 of them take mathematics, 36 chemistry and 30 take physics. The number of students who take chemistry and physics are 16, those who take chemistry and mathematics are 6, the number of students who take physics only are 8 while those who take chemistry only are 16.

   i) Represent the information in a Venn diagram (8 Marks)

   ii) Determine the number of students who take exactly two subjects (1 Mark)

   iii) Determine the number of students who take exactly one subject (1 Mark)

c) Find the square-roots of the complex number $z = 1 + i$ leaving your answer in Cartesian form (7 Marks)