MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

FOURTH YEAR **SECOND** SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE APPLIED STATISTICS WITH PROGRAMMING

AMS 409 – STOCHASTIC PROCESSES II

DURATION: 2 HOURS

DATE: 23/04/2019

TIME: 9-11am

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Define each of the following examples of a stochastic process

i) A symmetric simple random walk. 2marks

ii) A compound Poisson process. 2marks

iii) A birth and death process. 3marks

b) Patients arrive at the doctor's office according to a Poisson process with rate $\lambda = \frac{1}{10}$ minutes. The doctor will not see a patient until at least three patients are in the waiting room.

i) Find the expected waiting time until the first patient is admitted to see the doctor 4marks

ii) What is the probability that nobody is admitted to see the doctor in the first hour? 4marks

c) Is the process with the following transition matrix irreducible?

$$\begin{pmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 2/3 & 0 & 1/3 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$$ 5marks

d) From the above, what is the stationary distribution of the process? 5marks

e) Let x(t) be a poisson process with parameterλ. Find

i) E [X^2(t)] 2marks

ii)E[X(t) – X(s)]^2 , $for\ t > s$ 3marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Consider a market process with state space S = (0, 1, 2) and transition matrix, P

$$\begin{pmatrix} p & q & 0 \\ 1/2 & 0 & 1/2 \\ p-1/2 & 7/10 & 1/5 \end{pmatrix}$$

i) What can you say about values of p and q. 2marks

ii) Calculate the transition probabilities. 4marks

iii) Draw the transition graph for the process represented by P. 4marks

b) A system consists of 3 machine and 2 repairmen. At most 2 machines can operate at anytime. The amount of time that an operating machine works before breaking down is exponentially distributed with mean 5 hours. The amount of time that it takes a single repairman to fix a machine is exponentially distributed with mean 4 hours. Only one repairman can work on a failed machine at any given time. Let $X(t)$ be the number of machines in operating condition t time t.

i) Calculate the long run probability distribution of $X(t)$. 5maks

ii) If an operating machine produces 100 units of output per hour, what is the long run output per hour from the factory? 5marks

QUESTION THREE (20 MARKS)

Two gamblers play the following game. A fair coin is flipped, if the outcome is heads, player A pays player B \$10, and if the outcome is tail, player B pays A \$1. The game is continued until one of the players goes broke. Suppose that initially player A has \$1 and player B has \$2, so as a total of \$3 is up for grabs. Let X_n denote the number of dollars held by player A after n trials.

i) Sketch the transition diagram for X_n and give the one step transition probability matrix p. 3marks

ii) Use the state transition diagram to help you show that for n even (i.e. n = 2k).

$$P_n^{(n)} = (\tfrac{1}{2})^n \ \ for\ i = 1,2$$

$$P_{10}^{(n)} = \tfrac{2}{3}\left[1 - \left(\tfrac{1}{4}\right)^k\right] = P_{23}^{(n)}$$ 5marks

iii) Find the n-step transition probability matrix for n even using part (ii). 4marks

iv) Find the limit of P^n as $n \to \infty$ 4marks

v) Find the probability that player A eventually wins. 4marks

QUESTION FOUR (20 MARKS)

a) Define what a counting process is and state two properties. 3marks

b) Assume that certain events (say, power surges) occur as a poisson process with rate 3 per hr. These events cause damage to certain system (say, a computer), thus a special protecting unit has been designed. That unit now has to be removed from the system for 10 minutes for service.

i) Assume that a single event occurring in the service period will cause the system to crash, what is the probability that the system will crash? 2marks

ii)) Assume that the system will survive a single event, but two events occurring in the service period will cause it to crash. What is now the probability that the system will crash? 2marks

iii) Assume that crash will not happen unless there are 2 events within 5 minutes of each other. Compute the probability that the system will crash 3marks

iv) Solve (iii) assuming that the protective unit will be out of the system for time which is exponentially distributed within expectation 10 minutes. 3marks

c) Consider a Markov chain with only two states S = {0, 1} and the transition matrix

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Determine the stationary distribution of this chain. 4marks

d) A two state Markov chain has transition matrix

$$\begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix} \end{array}$$

If the process is in state 0, at time 0, explain how you would simulate a series of observations from this process. 3marks