MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

FOURTH YEAR **SECOND** SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE

AMS 404 – MULTIVARIATE METHODS

DURATION: 2 HOURS

DATE: 17/04/2019

TIME: 9-11

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) State and explain any four objectives of scientific observations based on multivariate data

4marks

b) Consider the data matrix $\boldsymbol{X^T} = \begin{pmatrix} 14 & 13 & 11 & 12 & 11 \\ 12 & 10 & 12 & 15 & 18 \end{pmatrix}$ Test at α=0.1 the hypothesis

$H_0: \mu^T = \begin{pmatrix} 16 \\ 11 \end{pmatrix}$

against $H_0: \mu^T \neq \begin{pmatrix} 16 \\ 11 \end{pmatrix}$ 7marks

c) Explain briefly the multivariate analysis concepts behind the following

i. Canonical correlation analysis 2marks

ii. Cluster analysis 2marks

iii. Discriminant analysis 2marks

d) Given that $X \sim N_3(\mu, \Sigma)$ with $\mu = \begin{pmatrix} 60 \\ 48 \\ 24 \end{pmatrix}, \Sigma = \begin{pmatrix} 9 & 6 & \frac{12}{5} \\ 6 & 36 & -12 \\ \frac{12}{5} & -12 & 16 \end{pmatrix}$, Find:

i. The conditional variance of X_1 when given X_2 and X_3 4marks

ii. The multiple correlation coefficient between X_1 and (X_2, X_3) 2marks

e) Give an expression for multivariate normal distribution and explain the meaning of non-singular multivariate normal distribution 3marks

f) Given that $\boldsymbol{X} \sim N_3 \left(\mu = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}, \Sigma = \begin{pmatrix} 5 & 5 & 0 \\ 6 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \right)$Determine whether:

i) X_1 and X_2 are independent 2marks

ii) (X_1, X_2) and X_3 are independent 2marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Define multivariate analysis 2marks

b) Given that $\boldsymbol{X} \sim N_3(\mu, \Sigma)$ with $\mu = \begin{pmatrix} 23 \\ 44 \\ 27 \end{pmatrix}, \Sigma = \begin{pmatrix} 100 & \frac{110}{3} & 20 \\ \frac{110}{3} & 121 & 44 \\ 20 & 44 & 64 \end{pmatrix}$

Find:

i. The distribution of Y where $Y_1 = X_1 - X_2 = +X_3$

$$Y_2 = 2X_2 = -X_3$$ 4marks

ii. The regression function of X_1 onX_2 and X_3 4marks

iii. The distribution of X_1given thatX_2= 39 and $X_3 = 29$ 4marks

iv. The partial correlation coefficient between X_1 and X_2 for fixed values of $X_3$and that of X_1 and X_3 for fixed values of X_2 6marks

QUESTION THREE (20 MARKS)

a) State the assumptions behind the idea of using MANOVA 4marks

b) Observation on three responses are collected for two treatments as shown in the table

Treatment	1	1	1	1	1	2	2	2
X1	8	9	5	4	4	6	7	5
X2	15	14	12	9	10	6	8	10
X3	4	5	4	4	3	9	8	7

Find:

i. The matrix of sum of squares due to treatment. 5marks

ii. The matrix of residual sum of squares. 5marks

iii. The Wilk's lambda statistic and use it to test the hypothesis that there is no treatment effects (use $\alpha = 0.05$). 6marks

QUESTION FOUR (20 MARKS)

a) Let$X \sim N(\mu, \Sigma)$ be a trivariate normal random vector with sample variance covariance matrix –

$$S = \begin{pmatrix} 4 & 0 & 3 \\ 0 & 9 & 0 \\ 3 & 0 & 25 \end{pmatrix}$$

Find

i) The first two principle components and the total variance explained by the components

14marks

ii) The correlation between the first principle component and the second variable 2marks

b) Give a brief overview and the objectives of principle components 4marks