MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

THIRD YEAR **SECOND** SEMESTER EXAMINATION

AMS 331 – PROBABILITY AND STATISTICS IV

DURATION: 2 HOURS

DATE: 25th April 2019

TIME: 2-4pm

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Define the following terms

i. Random variable 1mark
ii. Random vector 1mark

b) State the chebychers inequality 2marks

c) Consider the following six observations of two variables

X_1	X_2
100	20
95	18
110	22
100	20
100	21
95	19

Find the sample mean vector and the sample covariance matrix for this data 7marks

d) Find the generating function A(s) for the following sequence $\{a_k\} = \frac{1}{k!} \quad k = 1, 2, ...$
4marks

e) Let $X \sim N_4\left(\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\begin{bmatrix}4 & 1 & 2 & 4\\1 & 4 & 2 & 1\\2 & 2 & 16 & 1\\4 & 1 & 1 & 9\end{bmatrix}\right)$

Find the marginal distribution of $\begin{bmatrix}X_2\\X_4\end{bmatrix}$ 5marks

f) Find the probability generating function for the following distribution

$P(X = x) = p(1-p)^2 \quad x = 0, 1, 2 ...$
3marks

g) Prove that the covariance matrix $\sum X$ is positive semidefinite, that is, $a^T \sum X\, a \geq 0$ for every real vector a. 4marks

h) Consider the following matrix $\sum = \begin{bmatrix}4 & 1 & 0\\1 & 3 & 0\\0 & 0 & 2\end{bmatrix}$

Find the covariance matrix for $X_1 = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ and X_3 and hence show if they are independent.

3marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Find the generating function A(s) for the following sequence $\{a_k\} = k^4, \quad k = 0, 1, 2, \ldots$

10marks

b) Let the probability density function of a random variable X be

$$f(x) = \begin{cases} 630x^4(1-x)^4 & if\ 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

i. Find the exact value of $P(|X - \mu| \leq 2\sigma)$ 8marks

ii. What is the approximate value using Chebychers inequality. 2 marks

QUESTION THREE (20 MARKS)

a) Let $X = [X_1, X_2, X_3]'$ be a three – dimensional random vector. Suppose that X, has a mean vector $\mu = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and a covariance matrix $\sum = \begin{bmatrix} 11 & -6 & 2 \\ -6 & 10 & -4 \\ 2 & -4 & 6 \end{bmatrix}$

i. Define $Y = [Y_1, Y_2,]'$ with $Y_1 = X_1 + X_2$ and $Y_2 = X_1 + 2X_3$

Find the mean vector and the covariance matrix of Y

ii. Consider the following linear combinations

$Z_1 = X_3$

$Z_2 = X_1 + X_2 + X_3$

Find the mean vector and the covariance matrix of Z 12marks

b) Let $Y = X_1 + X_2 + \ldots + X_{15}$ be the sum of a random sample of size 15 from the distribution

whose density function is $f(x) = \begin{cases} \frac{3}{2}x^2 & if -1 < x < 1 \\ 0 & elsewhere \end{cases}$

What is the approximate value of $P(-0.3 \leq Y \leq 1.5)$ when one uses the central limit theorem

8marks

QUESTION FOUR (20 MARKS)

a) Suppose that X_1, X_2, X_3 *and* X_4 and normally distributed with

$$\mu = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \; and \sum = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$$

i. Find the marginal distribution of $\begin{bmatrix} X_1 \\ X_3 \end{bmatrix}$

5marks

ii. Find the conditional distribution of (X_1, X_2) given (X_3, X_4).

13marks

b) State two properties of the covariance matrix. 2marks