MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

SECOND YEAR **SECOND** SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE AND ACTUARIAL SCIENCE

AMS 330 – PROBABILITY AND STATISTICS III

DURATION: 2 HOURS

DATE: 16/04/19

TIME: 9-11

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Let the joint probability density function of X_1 and X_2 be given by

$$f(x_1, x_2) = \begin{cases} 2x_1 + x_2, & 0 < x_1 < 1, \ 0 < x_2 < 1 \\ 0, & Otherwise \end{cases}$$

Determine whether X_1 and X_2 are stochastically independent. (4marks)

b) Let X be a continuous random variable with pdf given by

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1, \\ 0, & Otherwise \end{cases}$$

Find the probability density function of $Y = X^3$ (4marks)

c) Two random variables X and Y have a joint probability distribution, $P(X = x, Y = y)$ given by

		Y		
		0	1	2
X	0	$\frac{3}{24}$	$\frac{1}{24}$	$\frac{1}{24}$
	1	$\frac{2}{24}$	$\frac{5}{24}$	$\frac{2}{24}$
	2	$\frac{7}{24}$	$\frac{1}{24}$	$\frac{2}{24}$

i. What are the marginal probability distribution of X and Y. (6marks)

ii. What are the values of $E(x)$ and $Var\ (y)$. (6marks)

d) Karl Pearson analyzed 1050 pairs of heights of mothers, X, and the adult height of their daughters, Y, (in inches). Assuming the parameters in a bivariate normal model are

$\mu_x = 68.5 \ \ \mu_y = 69.7, \quad \sigma_x = 3.74, \ \ \sigma_y = 3.71, \ \ \rho_{xy} = 0.501$

If the bivariate normal distribution is appropriate for mother who is six feet tall X is equal to 77, find the expected height of her daughter and the standard deviation of the height of the daughter.

(5marks)

e) Write an R- Code to generate 1001 random (X, Y) pairs for the bivariate normal model with parameters

$\mu_x = 67.5$ $\mu_y = 68.7$, $\sigma_x = 2.74$, $\sigma_y = 2.81$, $\rho_{xy} = 0.5$ (3marks)

f) State any two properties that a bivariate joint probability function of discrete random variables X and Y should satisfy. (2marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) The joint density function of X_1 and X_2 is given by

$$f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & x_1 > 0, x_2 > 0 \\ 0, & elsewhere \end{cases}$$

Find the density function of $Y = \frac{X_1.}{X_1+X_2}$ (7marks)

b) Let X and Y be a joint function of a bivariate normal distribution for a random variable X and Y.

i. Write down this density function defining all the parameters. (2marks)

ii. Find the marginal density of X. (5marks)

iii. Find the conditional density of Y given X. (6marks)

QUESTION THREE (20 MARKS)

a) The joint distribution of two random variables X and Y is given by

		Y		
		-4	2	7
X	1	1/8	1/4	1/8
	5	1/4	1/8	1/8

Compute the

a) $E(X)$ and $E(Y)$ (2marks)

b) $E(XY)$ (2marks)

c) σ_X and σ_y (2marks)

d) $Cov\ (XY)$ (2marks)

e) $\rho_{xy}\ (X,Y)$ (2marks)

b) The joint density function of X_1 and X_2 is

$$f(x_1,x_2) = \begin{cases} h(x_1+x_2), & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0, & Otherwise \end{cases}$$

i. Find h (3marks)

ii. The marginal density of X_2

iii. The conditional expectation of X_1, given $X_2 = \frac{1}{3}$ (4marks)

QUESTION FOUR (20 MARKS)

a) Let X and Y be independent random variables such that $X \sim N(0,1)$ and $Y \sim X_r^2$. Given that the random variable.

$$T = \frac{X}{\sqrt{\frac{Y}{r}}}$$

show that T follows a t-distribution with r-degrees of freedom. (8marks)

c) Let U_1 and U_2 be independent random variables where U_1 has a chi-square distribution with v_1 degrees of freedom and U_2 has a chi-square distribution with v_2 degrees of freedom.

Derive the probability distribution function of the random variable $F = \frac{\frac{U_1}{v_1}}{\frac{u_2}{v_2}}$ (12marks)