MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

FIRST YEAR **SECOND** SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

AMS 107 – PROBABILITY AND DISTRIBUTION THEORY I

DURATION: 2 HOURS

DATE: 26-04-2019

TIME: 9-11AM

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) A continuous random variable X has a moment generating function M(t) = e^{t^2+t}. Determine the mean and variance of X (4marks)

b) A random variable x has probability distribution P(X) given by

X	0	1	2	3
P(X)	0.2	0.3	a	b

Given that E(X) = 1.4, determine the probability a and b and hence the variance of X . (6marks)

c) State any two properties that a probability function of discrete random variable X should satisfy. (2marks)

d) Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} (3-x)(1+x), & 0 \le x \le 3 \\ 0, & Otherwise \end{cases}$$

Find the E(X) (4marks)

e) Let the joint pdf of X and Y be given by

$$f(x,y) = \begin{cases} x+y, & 0 < x < 1, 0 < y < 1 \\ 0, & Otherwise \end{cases}$$

Determine whether X and Y are stochastically independent (4marks)

f) The joint probability function of two discrete random variables x and y is given by

$$f(x,y) = \begin{cases} k(x+y), & 0 \le x \le 2, 0 \le y \le 3 \\ 0, & Otherwise \end{cases}$$

Find

i. The value of k (3marks)
ii. P(x ≥ 1, y ≤ 2) (3marks)

g) If X is a discrete random variable with a and b being constant. Show that

$Var\ (ax+b) = a^2 var(x)$ (4marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

The joint probability density distribution function of X_1 and X_2 is

$$f(x,y) = \begin{cases} k(x_1 + x_2), & 0 < x_1 < 2, 0 < x_2 < 3 \\ 0, & Otherwise \end{cases}$$

Find

i. The value k (3marks)

ii. E(x_1), E(x_2), V(x_1), V(x_2) [10marks]

iii. Cov (x_1, x_2) [4marks]

iv. Correlation between X_1 and X_2 [3marks]

QUESTION THREE (20 MARKS)

a) Let X be a random variable with generating function G (t). Show that the mean (x) and variance (x) are given by $G'(1)$ and $G''(1)$ + $G'(1)$ - [$G'(1)$]^2 respectively where the dashes means differentiation with respect to t. (5marks)

b) The moment generating function of a gamma distribution of the second order is given by

$$m(t) = (1 - \beta t)^{-\alpha}, \qquad \alpha>0, \beta>0$$

Use moment generating function to determine the mean and variance of the distribution (6marks)

c) Let X be a continuous random variable having pdf f(x) given by

$$f(x) = \begin{cases} 3x^4, & 0 \le x \le 1 \\ 0, & Otherwise \end{cases}$$

Determine the

i. Pdf of continuous random variable Y where Y = X^3 (3marks)

ii. $P(\frac{1}{2}<X<1)$ (3marks)

iii. $P(\frac{1}{8}<Y<1)$ (3marks)

QUESTION FOUR (20 MARKS)

a) A torch battery has a life time of T hours with a pdf

b) $f(t) = \begin{cases} \frac{1}{100} e^{-\frac{t}{100}}, & t > 0 \\ 0, Otherwise \end{cases}$

Determine the probability that the battery

i. Fails before 30 hrs (2marks)

ii. Life is between 35 and 50 hrs (2marks)

iii. The life is greater than mean life (2marks)

c) A die is rolled until a six appears. Let X be the number of trials until a six appears. Find the mean of X using probability generating function . (6marks)

d) A continuous random variable X is uniformly distributed over the internal [a,b]. Use the moment generating function to derive the mean and variance of X. (8marks)