MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

FOURTH YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMM 414 – NUMERICAL ANALYSIS II

DURATION: 2 HOURS

DATE: 25/4/2019

TIME: 9-11 A.M.

Instructions to candidates:

1. Answer question One and Any Other Two questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) The function f(x)=Cos x is defined on the interval [1,3]

   i. Obtain the Lagrange linear interpolating polynomial in this interval (3 marks)

   ii. Find the bound on the truncation error (3 marks)

   iii. Approximate values of f(1.5) and f(2.5) (2 marks)

b) For the following data, calculate the differences and obtain the backward difference polynomial interpolate at x=0.45 (5 marks)

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.00	2.28

c) Obtain the least squares polynomial approximation of degree one for f(x)=$x^{1/3}$ on (0,1) with w(x)=1 (5 marks)

d) Solve the following system of equations using Cramer's rule (5 marks

$$\begin{bmatrix} 2 & 2 & 1 \\ 4 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

e) Give the corresponding FORTAN expressions for the given Algebraic expressions

   i. $a = x + \frac{y}{z} - r^2$ (1 mark)

   ii. $b = \frac{x+y}{z} + rt$ (1 mark)

   iii. c = (xy)(z+2) (1 mark)

f) In numerical integration, find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ using trapezoidal rule. Obtain a bound for the errors. The exact value of I=In2=0.693147 (4 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Solve the system of equations

$4x_1 + x_2 + x_3 = 2$

$x_1 + 5x_2 + 2x_3 = -6$

$x_1 + 2x_2 + 3x_3 = -4$

Using the Jacobi iteration method, take the initial approximation as $x^{(0)} = (0, 0, 0)^T$ and perform three iterations. The exact solution is $x_1=1$, $x_2=-1$, $x_3=-1$ (7 marks)

b) Prove that the Chebyshev polynomial of order four is given by;

$T_4(x)=8x^4-8x^2+1$ (5 marks)

c) Using the nodes $x_0=2$, $x_1=4$, find the second Lagrange interpolating polynomial for $f(x)=\frac{1}{x^2}$ (4 marks)

d) For the following data, calculate the difference and obtain the backward difference polynomial.interpolate at x=2. (4 marks)

x	1.5	2.5
f(x)	3	5.5

QUESTION THREE (20 MARKS)

a) By applying partial pivoting if necessary, solve the system of equations using Gauss elimination method. (6 marks)

$5x_1 + x_2 + x_3 – 2x_4 =-12$

$4x_1 + 2x_2 + 6x_3 + x_4 = 8$

$3x_1 +2x_2 +2x_3 +5x_4 = 7$

$X_1 + 3x_2 +2x_3 – x_4 = -5$

b) Derive the trapezoidal method of numerical integration using the method of undetermined coefficient. (6 marks)

c) (i) When is a polynomial p(x) referred to as an interpolating polynomial? (1 mark)

(ii) Give the Taylor series expansion for the function f(x) about a point x_0, $x_0 \in [a,b)$

(2 marks)

d) Using the Lagrange formula, find the unique polynomial of degree 2 which fits the given data. (5 marks)

x	0	2	4
f(x)	1	8	16

QUESTION FOUR (20 MARKS)

a) Obtain the Taylor series approximation about x=1 up to second degree terms for the function. (6 marks)

$$F(x) = \frac{1}{1+x^2}$$

b) Find a bound on the error if this approximation is to be used in (1, 2). (4 marks)

c) (i) Apart from the Euler's method, give any other two methods that can be applicable in determining the numerical solution y(t) of the problem;

$\frac{dy}{dt} = f(t,y)$ for $a \leq t \leq b$ (2 marks)

(ii) Find the approximation to the initial value problem

$y^1 = -y + 1$ $\quad 0 \leq t \leq 1$

y(0) = 0 N=10 ti=a+ih h=0.025 for $W_1, W_2, W_3, \ldots$ (6 marks)

d) For Legendre polynomials show that;

$$\int_{-1}^{1} Pm^{(x)}\, Pn^{(x)} dx = 0$$

when m=0 and n=1 (2 marks)

QUESTION FIVE (20 MARKS)

a) The following values of temperature x in degrees Celsius and the corresponding function f(x); f(x)=sinx + cosx governing heat in kilojoules are given by:

x	10^0	20^0	30^0
F(x)	1.1585	1.28171	1.3660

Construct the quadratic interpolating polynomial that fits the data. Hence find $f(\frac{\pi}{2})$. Compare with the exact value. NB: your answer should be in radians. (8 marks)

b) Solve the system equations

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Using the cholesky method (5 marks)

c) Use the simple method to find the maximum value of p = x + 4y subject to;

$-x + 2y \leq 6$

$5x + 4y \leq 40$

$x, y \geq 0$

Perform two iterations only (7 marks)