MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

FOURTH YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE

AMM 410 –PARTIAL DIFFERENTIAL EQUATIONS 11

DURATION: 2 HOURS

DATE:

TIME:

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Solve the partial differential equation $xr+p = 9x^2y^3$ where $r = \frac{\partial^2 z}{\partial x^2}$ *and* $p = \frac{\partial z}{\partial x}$ 6mks

b) Classify the second order partial differential equation

$U_{xx} + 2yU_{xy} + xU_{yy} - U_x + U = 0$ 4 mks

c) Find the characteristics of the partial differential equation

$xU_{xx} + (x - y)U_{xy} - yU_{yy} = 0$ 4 mks

d) Separate the partial differential equation $t^3u_{xx}+x^3u_{tt} = 0$ into ordinary differential equations of x and t. 5mks

e) i. Show that the linear partial differential equation $\frac{\partial^2 U}{\partial \theta^2} - \frac{\partial^2 U}{\partial t^2} = 0$ admits the solution $U(t, \theta) = Sin\ \theta\ Sin\ t.$ 3mks

ii. Does this solution satisfy the boundary condition $U(t, \pi) = 0$ and the initial condition $U(0, \theta) = 0$? 2mks

f) Use separation of variables to reduce the partial differential equation

$$\frac{\partial U}{\partial t} = c\frac{\partial^2 U}{\partial x^2}$$

$U(x, 0) = f(x),\ \frac{\partial U}{\partial x}(0, t) = 0, \frac{\partial U}{\partial x}(L, t) = 0$ where c is a constant into ordinary differential equations. 6mks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Solve the partial differential equations $s = 2x + 2y\ where\ s = \frac{\partial^2 Z}{\partial x \partial y}$ 4mks

b) Classify and solve the partial differential equation $u_{xx} + u_{xy} - 2u_{yy} = 0$ by assuming solutions of the form u = f(mx+y) 6mks

c) Reduce the partial differential equation $u_{xx} + 5u_{xy} + 6u_{yy} = 0$ to its canonical form and find the general solution 10mks

QUESTION THREE (20 MARKS)

a) Classify the one dimensional wave equation $U_{tt} - c^2 U_{xx} = 0$ where c is a constant 4mks

b) Reduce the equation in 3 (a) to its normal form and find the solution of the one dimensional wave equation 10mks

c) Use separation of variables to reduce the equation in 3(a) to corresponding ordinary differential equations 6mks

QUESTION FOUR (20 MARKS)

a) Given the heat equation $\frac{\partial U}{\partial t} = k\frac{\partial^2 U}{\partial x^2}$

$U(x,0) = f(x), U(0,t) = 0, U(L,t) = 0$, use separation of variables to reduce it to ordinary differential equations. 6mks

b) By use of the result in 4 (a) above, find a solution to the one-dimensional heat equation which satisfies the given boundary conditions 14mks