



# **MURANG'A UNIVERSITY OF TECHNOLOGY**

## **SCHOOL OF PURE AND APPLIED SCIENCES**

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

**FOURTH YEAR SECOND SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE AND BACHELOR OF SCIENCE IN MATHEMATICS & ECONOMICS**

AMM 403 – COMPLEX ANALYSIS 11

DURATION: 2 HOURS

DATE:

TIME:

**Instructions to candidates:**

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

**SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION**

**QUESTION ONE (30 MARKS)**

- a) Locate and name all the singularities of the function  $\frac{(z+3i)^5}{(z^2-2z+5)^2}$  5marks
- b) Use expansion method to find the residue at the origin of the function  $f(z) = z^{-2} \cot z$  5marks
- c) Evaluate  $\oint_c \frac{1-2z}{z(z-1)(z-2)} dz$  where  $c$  is the circle  $|z| = 3$  5marks
- d) Use Cauchy's residue theorem to evaluate the real integral  $\int_0^{2\pi} \frac{d\theta}{5-3\sin\theta}$  5marks
- e) Show that if the function  $f(z)$  is analytic and  $f'(z)$  is not equal to zero in a region  $R$ , then the mapping  $w=f(z)$  is conformal at all points of  $R$  5marks
- f) Show that the functions  $g(z) = \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$  and  $h(z) = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(2-i)^{n+1}}$  are analytic continuations of each other. 5marks

**SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION**

**QUESTION TWO (20 MARKS)**

- a) Evaluate  $\oint_c \frac{e^z}{z^2(z^2+2z+2)} dz$  where  $c$  is the circle  $|z| = 2$  10marks
- b) By use of residues of a complex function, evaluate the real integral  $\int_0^{2\pi} \frac{\cos 3\theta}{5-4 \cos \theta} d\theta$  10 marks

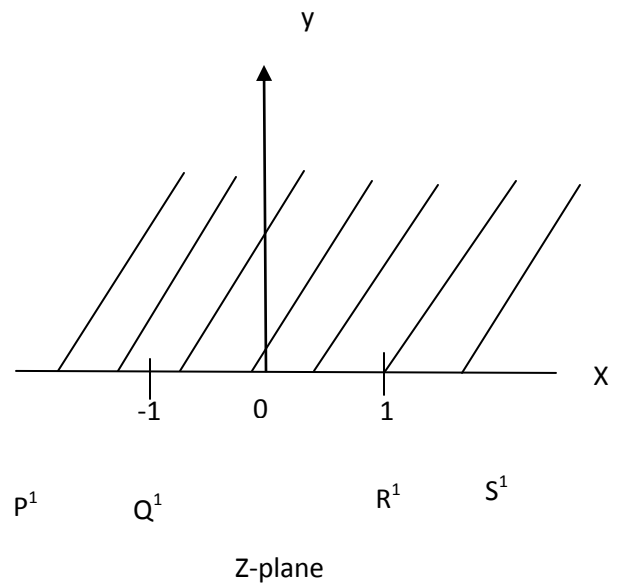
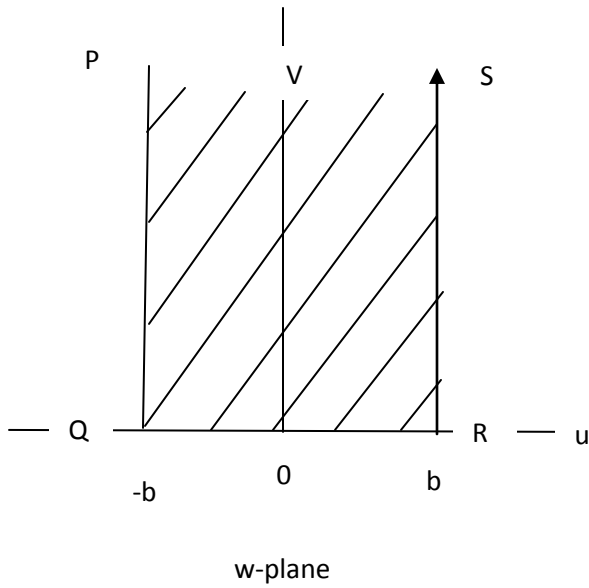
**QUESTION THREE (20 MARKS)**

- a) Evaluate  $\int_{-\infty}^{\infty} \frac{dx}{x^6+1}$  using Cauchy's residue theorem 12marks
- b) Show that the effect of two successive bilinear transformations is also a bilinear transformation 8marks

**QUESTION FOUR (20 MARKS)**

a) Determine the image in the w-plane of the circle  $|z| = 2$  in the z-plane under the transformation  $w = \frac{z+i}{z-i}$  10marks

b) Use Schwarz-Christoffel transformation to determine a function which maps the region in the w-plane shown on to the upper half of the z-plane



10marks