MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

THIRD YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE

AMM 316 – ORDINARY DIFFERENTIAL EQUATIONS 11

DURATION: 2 HOURS

DATE:

TIME:

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Show that the Wronskian of the functions $f(x) = e^{ax}\cos bx$ and $g(x) = e^{ax}\sin bx$, $b \neq 0$ is be^{2ax}

5marks

b) Write the following 4^{th} order differential eqution as a system of first order linear differential equations

$y^{iv}+3y^{ii}-\text{sin}ty^{i}+8y = t^2$: $y(0) = 1$, $y^{1}(0) = 2$, $y^{ii}(0) = 3$, $y^{iii}(0) = 4$ 5marks

c) Write the following first order system of linear differential equations in matrix form

$$X_1' = 4X_1 + 7X_2$$

$$X_2' = -2X_1 - 5X_2$$

4marks

d) Determine eigenvalues and corresponding eigenvectors of the matrix $A = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix}$ 5marks

e) Solve the second order non-linear differential equation $y^{ii} = -2t(y^{i})^2$ with initial conditions $y(0) = 2$, $y^{i}(0) = -1$ 6marks

f) Determine and classify the singular points of the differential equation

$$x^2(x-2)^2\frac{d^2y}{dx^2} + 2(x-2)\frac{dy}{dx} + (x+1)y = 0$$ 5marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Show that the functions $9\cos 2x$ and $2\cos^2 x - 2\sin^2 x$ are linearly dependent 5marks

b) Show that the Wronskian of the functions x^2 and xlnx is non-zero. Can these functions be independent solutions of an ordinary differential equation? If so determine this equation

15 marks

QUESTION THREE (20 MARKS)

a) Convert the differential equation $y^{ii}+y^{i}-6y = 0$, $y(0) = 0$, $y^{i}(0) = 5$ into a first order system of linear differential equations and write the system in matrix form. 5marks

b) Solve the following initial value problem.

$$\boldsymbol{X}' = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}\boldsymbol{X}; \quad \boldsymbol{X}(0) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$ 15marks

QUESTION FOUR (20 MARKS)

a) Reduce Bessel's equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ to its normal form. 5marks

b) Find the general solution to the following non-homogeneous system of linear first order differential equations $\boldsymbol{X}' = \begin{pmatrix} 2 & 3 \\ 4 & 3 \end{pmatrix}\boldsymbol{X} + t\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ 15marks