MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

THIRD YEAR **SECOND** SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE (Applied Statistics with Programming)

AMM 312 – METHODS

DURATION: 2 HOURS

DATE: 16/04/2019

TIME: 2:00 – 4:00 pm

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Given that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ find $\Gamma\left(-\frac{1}{2}\right)$ 3marks

b) Determine whether the function $f(x) = \begin{cases} x^{-3} & 0 < x < 2 \\ 0 & elsewhere \end{cases}$

is absolutely integral or not 3marks

c) Evaluate $\int_0^\infty \frac{e^{-4t}}{\sqrt{t}} dt$ 4marks

d) Prove that $\mathcal{L}\{\int_0^\infty f(u)du\} = \frac{1}{s}\mathcal{L}\{f(t)\}$ 5marks

e) Find the half range Fourier sine series representation for the function

$$f(t) = \begin{cases} 2t - 3 & 0 < t < 3 \\ f(t+6) & \end{cases}$$ 6marks

f) Use first principles to find the Laplace transform of the function $f(t) = H(t-3)e^{-4t}$ where $H(t)$ is the Heaviside unit step function 4marks

g) Find the Fourier transform of function $f(t) = e^{-2|t|}$ 5marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Find the Fourier cosine transform of the function defined by $f(t) = te^{-t}$ 6marks

b) Find the Fourier transform of the function $f(t) = \frac{1}{t^2+4}$ 6marks

c) Evaluate $\int_0^\infty \frac{e^{-2x} - e^{-3x}}{x} dx$ 8marks

QUESTION THREE (20 MARKS)

a) Use the convolution theorem to find $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s-2)}\right\}$ 6marks

b) A system at rest has a constant input $f(t) = 4$ applied when $t = 0$. The output is found to be $x(t) = 1 - cos\ 2t$. Find the impulse response function of this system 5marks

c) Use Laplace transforms to find the solution to the initial value problem

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 12x = 5e^{2t}, \quad x(0) = 0, \ x'(0) = 1$$ 9marks

QUESTION FOUR (20 MARKS)

a) Find the half-range Fourier cosine series representation for the function

$$f(t) = \begin{cases} 2t + 3 & 0 < t < 1 \\ f(t+2) & \end{cases}$$ 8marks

b) Find the Fourier series representation for a periodic function defined by

$$f(t) = \begin{cases} t - 2 & -2 < t < 0 \\ 2t + 1 & 0 < t < 2 \\ f(t+4) & \end{cases}$$ 12marks