MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE (Mathematics and Computer Science, YR 4) and BACHELOR OF SCIENCE (Mathematics and Economics, YR 3)

AMM 308 – COMPLEX ANALYSIS 1

DURATION: 2 HOURS

DATE: 23/04/2019

TIME: 2:00 – 4:00 pm

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Find the principal value of complex number $z = 2 - 5i$ 3marks

b) Given the complex number $z = 3 + 2i$, find

i. $ln\,(z)$ 3marks

ii. z^i 4marks

c) Show that the function defined by $f(z) = 2z^2 - 5z + 3$ is analytic 5marks

d) Given the function $w = (2x^2 - 2y^2 - 4x + 5) + i\,(4xy - 4y + 2)$, find $\frac{dw}{dz}$ 2marks

e) Find the residue of the function $f(z) = \frac{\tan z}{\left(z - \frac{\pi}{4}\right)^2}$ at $z = \frac{\pi}{4}$ 4marks

f) Use the Cauchy integral formula to evaluate $\oint_c \frac{e^z}{z^2 - 4}\,dz$ where c is the circle $|z - 1| = 2$

5marks

g) Classify the singular point of the function $f(z) = \frac{\cos z - 1}{z^2}$ 4marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Find the image of the line $x = 1$ under the transformation w = z^2 +3 5marks

b) Show that the function $u = x^3 - 3xy^2 - x^2 + y^2$ is harmonic hence find its conjugate harmonic function v such that $f(z) = u + iv$ is harmonic. 8marks

c) Find the image of the circle $|z| = 1$ under the transformation $w = \frac{z-1}{z-3}$ 7marks

QUESTION THREE (20 MARKS)

a) Determine the order of the pole of the function $f(z) = \frac{1}{(2\cos z - 2 + z)^2}$ at $z = 0$ 6marks

b))Find the Laurent series expansion for the function $f(z) = \frac{7z-11}{(z-1)(z-3)}$

in the annulus $1 < |z| < 3$ 8marks

c) Find a bilinear transformation that maps the points $z_1 = -1,\ z_2 = 0$ and $z_3 = 1$ onto $w_1 = -1,\ w_2 = i$, and $w_3 = 1$ respectively 6marks

QUESTION FOUR (20 MARKS)

a) Use the residue theorem to evaluate $\int_{-\infty}^{\infty} \frac{e^{\pi z}}{z^2+9}\, dz$ 5marks

b) Use Cauchy integral formula to evaluate

$$\oint_c \frac{e^z}{(z-1)(z+2)}\, dz$$

where c is the circle $|z| = 3$ 7marks

c) Evaluate $\int_0^{2\pi} \frac{1}{5 - 4\cos\theta}\, d\theta$ 8marks