



MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

**THIRD YEAR SECOND SEMESTER EXAMINATION FOR, BACHELOR OF
SCIENCE (Mathematics and Computer Science)**

AMM 207 – CLASSICAL MECHANICS

DURATION: 2 HOURS

DATE: 15/04/2019

TIME: 2:00 – 4:00 pm

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

Constants $g = 9.81\text{m/s}^2$

Gravitational constant

$G = 6.674 \times 10^{-11}$

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

- a) Differentiate between dimensions and units as used in dimensional analysis 2mks
- b) Define the term impulsive force 1mk
- c) State the three Newton laws of motion 3mks
- d) Find the work done in moving an objective along the vector $\vec{r} = 3\tilde{i} + 2\tilde{j} - 5\tilde{k}$ when the applied force is $\vec{F} = 2\tilde{i} - \tilde{j} - \tilde{k}$ 3mks
- e) A construction man drops a hammer from the floor of a building 30m high. If the hammer has a mass of 1.5 kg, calculate the velocity at which the hammer hits the ground (neglect air resistance). 3mks
- f) Find the instantaneous power applied to a particle of mass 3 units moving in a space curve whose position vector $\vec{r} = t^2\tilde{i} + (2t - 1)\tilde{j} + t^3\tilde{k}$ (5 Marks)
- g) A particle of unit mass moves in a force field defined by
- $$\vec{F} = (5t^2 + 3)\tilde{i} + (t^3 + 2t)\tilde{j} + (4t - 7t^2)\tilde{k} \text{ where } t \text{ denotes time.}$$
- i. Find the change in momentum of the particle from $t = 0$ to $t = 1$ 2mks
- ii. Given that the velocity of the particle when $t = 0$ is $\tilde{i} - 2\tilde{j} + 5\tilde{k}$, determine its velocity when $t = 1$ 3mks
- h) A man can throw a stone 90m vertically upwards. Find the greatest horizontal distance that he can throw the stone. 5mks
- i) Determine the force of gravitational attraction between the earth whose mass 5.98×10^{24} kg and a 100 kg mass object positioned at sea level whose distance from the center of earth is 6.38×10^6 m. 3mks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

- a) Given that the mass of the earth is 5.98×10^{24} kg and its distance from the sun is 1.5×10^{11} m and by assuming that its orbit is circular, calculate the magnitude of its angular momentum with sun as the origin (assume that the year has exactly 365 days). (3 Marks)

- b) A particle of mass 2 units moves along a straight line under the influence of uniform force of magnitude 12 units. Given the particle is initially at rest and at the origin, determine its velocity at any point x 6mks.
- c) A vertical spring has a stiffness factor 36 N per meter. At $t = 0$, a force $f(t) = 5 \cos 2t$ is applied to a 4.0kg mass which hangs in equilibrium at the end of the spring. Assuming no damping forces present,
- i. Set up a differential equation and the initial conditions describing this motion 2mks
 - ii. Solve the differential equation in part (i) above to find the position of the mass at any later time t sec. 9mks

QUESTION THREE (20 MARKS)

- a) Find the work done on a particle that moves from the point (0,1) to (2,5) along a curve $y = x^2 + 1$ under the influence of the force $\vec{F} = (2x^2 + y)\vec{i} + 3xy\vec{j}$ (7 Marks)
- b) A force field is defined by $\vec{F} = (6xy + 5z^2)\vec{i} + (3x^2 + 8yz)\vec{j} + (4y^2 + 10xz)\vec{k}$
- i) Show that \vec{F} is conservative 4mks
 - ii) Find the work done in moving a particle from the point A(1,0,2) to B(2,1,5) under the influence of the force \vec{F} 4mks
- c) A particle of mass m moves along the xy plane so that its position at any time t is given by $\vec{r} = a \cos \omega t \vec{i} + b \sin \omega t \vec{j}$ where a , b and ω are positive constants. Prove that the force acting on this particle is always directed towards the origin. 5mks

QUESTION FOUR (20 MARKS)

- a) A particle of mass m moves along the x -axis under the influence of a conservative force having the potential function $\phi(x)$. Given that the particle is located points x_1 and x_2 at a time t_1 and t_2 respectively.
- i) Prove that $\int_{x_1}^{x_2} \frac{1}{\sqrt{E - \phi(x)}} dx = \sqrt{\frac{2}{m}} (t_1 - t_2)$
where E is the total energy (6 Mk)

ii) Given that $\phi(x) = \frac{1}{2}kx^2$, where k is constant and that the particle starts at rest and at a point $x = a$, prove that $x = a \cos\left(\sqrt{\frac{2}{m}} t\right)$ (9 Marks)

b) A particle moving in a central force field located at $r = 0$ describes the spiral $r = e^{-\theta}$. Prove that the magnitude of the force is inversely proportional to r^3 5mks