MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

----------YEAR ------ SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE ----------------------

AMM 203 – VECTOR ANALYSIS

DURATION: 2 HOURS

DATE: 23/04/2019

TIME: 9.00 – 11.00 A. M

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Given that $\vec{U}_1 = 3\,\mathbf{i} + 4\,\mathbf{j} + 5\,\mathbf{k}$ and $\vec{U}_2 = \sin t\,\mathbf{i} + \cos t\,\mathbf{j}$, Find

i) $2\overrightarrow{U}_1 + 3\,\vec{U}_2$ — 2marks

ii) $\vec{U}_1 \cdot \vec{U}_2$ — 2marks

iii) The magnitude of $\vec{U}_1$ — 2marks

b) Find the equation of a line passing P(0,1,7) and Q(1,5.0) — 3marks

c) A particle travels 3 miles due North, the 5 miles due North East. Determine the resultant displacement — 3marks

d) A particle move so that its position vector is $\vec{r}(t) = \rho\sin\omega t\,\mathbf{i} + \rho\cos\omega t\,\mathbf{j}$, where ρ and ω are constants, and t is time in seconds. Calculate the arc length traced by the particle from $0 \leq t \leq 2\pi$ — 3marks

e) Consider a curve C which is parametrized as $\vec{R}(t) = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j}$
Prove that $\vec{R}(t)$ is perpendicular to $\vec{R}^1(t)$ — 2marks

f) Given that a vector force field $\vec{F}$ (x,y,z)= $(2x + \frac{3}{2}y^2)\,\mathbf{i} + 3xy\,\mathbf{j} + z\,\mathbf{k}$

i) Show $\vec{F}$ (x,y,z) is a conservative force field. — 2marks

ii) Calculate the scalar function ϕ(x,y,z) — 3marks

iii) Use results in f(ii) to calculate work done in moving a particle from (0,0,0) to (1,1,1) in the force field $\vec{F}$ (x,y,z) — 2marks

g) State Greens Theorem ` — 2marks

h) Evaluate

$$\iint_{\Omega} xy\,dxdy$$

where Ω is a rectangular region bounded by $0 \leq x - y \leq 1$ *and* $0 \leq 2x - y \leq 3$

4marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) A vector $\vec{A}$ has a magnitude of 6 units and $\vec{B}$ has a magnitude of 4 units. Find the angle between $\vec{A}$ and $\vec{B}$ if the **magnitude** of their sum is maximum 6marks

b) Find the $div\,\vec{V}$ and $Curl\,\vec{V}$ where $\vec{V}$ = $4xy$ **i** + yz **j** + x **k** 4marks

c) Find the work done in moving a particle from (0,0,0) to (1,1,1) in the force field

$\vec{F}$ = $(2x + y^2)$ **i** $- 3xy$ **j** + z **k**

i) If the path C is a straight line 5marks

ii) If the path is parametrized as

$$\begin{cases} x = t \\ y = t^2 \\ z = t^3 \end{cases}$$

5marks

take $\vec{r}$ = $x(t)$ **i** - $y(t)$ **j** + $z(t)$ **k**

QUESTION THREE (20 MARKS)

a) i) Determine the magnitude and the direction of the vector

$\vec{A}$ = 3 **i** - 4 **j** + 5 **k**

4marks

ii) Find a vector $\vec{B}$ of magnitude 100 units and travelling in the opposite direction of $\vec{A}$ in (i) above 3marks

iii) Find $\overrightarrow{A}$ - 2 $\overrightarrow{B}$ 2marks

b) Given two vectors $\vec{V}_1$ and $\vec{V}_2$ of magnitude 60 and 80 units, intersect at an angle of 60^0, find their dot products 3marks

c) Show that vectors

$$\begin{cases} \overrightarrow{A} = 3\,\mathbf{i} - 2\,\mathbf{j} + \mathbf{k} \\ \overrightarrow{B} = \mathbf{i} - 3\,\mathbf{j} + 5\,\mathbf{k} \\ \overrightarrow{C} = 2\,\mathbf{i} + \mathbf{j} - 4\,\mathbf{k} \end{cases}$$

For a right angled triangle 5marks

d) Prove that if $\vec{a}$ and $\overrightarrow{b}$ are non- collinear, and $\vec{a}\ x$ - $\vec{b}$ y $= 0$ then $x = y = 0$ where $x, y \in \mathbb{R}$

3marks

QUESTION FOUR (20 MARKS)

a) Use Greens theorem to evaluate

$$\oint_c (3x^2 + y)dx + (2x + y^3)dy$$

where $c \equiv x^2 + y^2 = 0$ 5marks

b) Calculate

$$\iint_R (x^2 + y^2)dxdy$$

where R is rectangle bounded by $\begin{cases} 0 \le x \le 4 \\ 0 \le y \le 2 \end{cases}$ 4marks

c) Given that $\overrightarrow{R}$(t) = $3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 4t\,\boldsymbol{k}$

i) $\tilde{T}$ 2marks

ii) $\widehat{N}$ 2marks

iii) $\hat{\beta}$ 2marks

iv) $\hat{\kappa}$ 2marks

v) $\hat{\rho}$ 2marks

d) State the Stokes theorem 1mark