MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

FIRST YEAR **SECOND** SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE (COMPUTER SCIENCE)

AMM 201 – LINEAR ALGEBRA 1

DURATION: 2 HOURS

DATE: 25/04/19

TIME: 2.00-400PM

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Give the meaning of the following terms and in each case give an example

i) A null matrix 2marks

ii) Scalar Quantity 2marks

iii) Linear independent vector 2marks

b) Give three ways of representing vectors in 2-dimensional form. 3marks

c) If V= -2i +j+4k and U =3i-2j+10k, calculate

i) V.U 3marks

ii) VxU 3marks

d) i) Define a vector norm 1mark

ii) Give two most commonly used vector norms 2marks

e) State and prove Triangle inequality theorem 3marks

f) If matrix $A = \begin{pmatrix} 2 & 2 \\ 4 & 3 \end{pmatrix}$; Derive any three echelon forms of the above matrix 3marks

g) i. State the Gram-Schmidt orthogonalization theorem. 2marks

ii. Perform the Gram-Schmidt process on this basis for $\Re^2$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, ensure that the resulting vectors are orthogonal and hence turn the orthogonal bases into orthonormal bases.

4marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Find the inverse of the linear mapping $M:(x,y)\rightarrow(x,\ 2x+y)$ and use it to find the point whose image is (2,1) under M. 5marks

b) Suppose the rows of a real 3x3 matrix A are interpreted as the components in a given basis of three component vectors a, b, and c. Show that one can write the determinant $|A| = a \bullet (b \times c)$.

3marks

c) Prove the cosine rule using the dot product. 5marks

d) The matrix for rotation through $0°$ about the origin is T = $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\ \theta \end{pmatrix}$

i) Find the image P' of the point $P(\sqrt{2},\frac{1}{\sqrt{2}})$ when it is rotated through 45^0 about the origin.

3marks

ii) Find the point Q whose image $Q'(6,2)$ is obtained by a transformation under the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ followed by a rotation through 90^0 about the origin. 4marks

QUESTION THREE (20 MARKS)

a) Four non-coplanar points A, B, C, D are positioned such that the line AD is perpendicular to BC and BD is perpendicular to AC. Show that CD is perpendicular to AB. 7marks

b) By using $\vec{a}$, $\vec{b}$ and $\vec{c}$ as position vectors and λ *and* μ as arbitrary scalars, give the three algebraic properties of vectors. 3marks

c) By the use of a diagram give the two ways that can enhance the understanding of vector sum geometrically. Use $\vec{V}$ and $\vec{W}$ to represent displacements. 5marks

d) Find the determinant of the matrix

$$A=\begin{pmatrix} 2 & 4 & 3 \\ 1 & -2 & -2 \\ -3 & 3 & 2 \end{pmatrix}$$ 3marks

e) Define a vector space 2marks

QUESTION FOUR (20 MARKS)

a) A triangle has vertices O(0,0), Q(4,2) and R(6,-2). Find the value of $\angle QOR$ 5marks

b) For the following vectors $A=\begin{pmatrix} k \\ 1 \\ 1 \end{pmatrix}$ and $B=\begin{pmatrix} 4 \\ 3 \\ 4 \end{pmatrix}$ Find k so that the two vectors are perpendicular. 4marks

c) Determine the rank of the matrix, $A=\begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{pmatrix}$ 5marks

d) Give the definition of the following and give an example of each.

i) A subspace 2marks

ii) A spanning set 2marks

iii) Dimension 2marks

QUESTION FIVE (20 MARKS)

a) i. Give the matrices A and B representing the following transformations respectively;

1. A reflection in the line $x+y=0$ 1mark
2. $(x,y)\rightarrow\left(\frac{\sqrt{3}}{2}x-\frac{1}{2}y,\frac{1}{2}x+\frac{\sqrt{3}}{2}y\right)$ 1mark

ii. Find the image of the point (2,4) under the transformation BA 3marks

iii. Find the matrix B^3 and identify which transformation it represents. 3marks

iv. If $C = \begin{pmatrix} -10 & 5 \\ -6 & 3 \end{pmatrix}$ and I is the 2×2 unit matrix, Find the matrix X such that $(A + C)X = I$ 4marks

b) By the use of the matrices; $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ where a_{ij} and b_{ij} Take distinct values, show that matrix multiplication is not commutative. 4marks

c) For the following bases in $\Re^2$; $M = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $N = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, Find the change of basis matrix. 4marks

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