



MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, MATHEMATICS AND ECONOMICS AND APPLIED STATISTICS WITH PROGRAMMING

AMM 109 – ANALYTICAL GEOMETRY

DURATION: 2 HOURS

DATE: 16/4/2019

TIME: 2- 4 PM

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

- a) Determine the distance between the point $P(-3,4)$ and the line $2y+3x-7 = 0$ 3marks
- b) Analyse the conic $x^2-y^2-4x+8y-21= 0$ 3marks
- c) Find the coordinates of a point $Q(x,y)$ which divides the line segment \overline{AB} externally in the ratio $m:n$ 4marks
- d) Convert the Cartesian coordinates $(-9,30)$ to polar coordinates 3marks
- e) Find the center and radius of circle $4x^2+4y^2+2x-10y+3 = 0$ 3marks
- f) Find the equation of the tangent at the point $(-1,2)$ on the ellipse $(x+1)^2+4(y-1)^2 = 4$ 4marks
- g) Find the equation of a parabola whose focus is $f(5,0)$ and whose equation of the directrix is $x+5 = 0$ 3marks
- h) Given that $Z = \left(2, \frac{4}{3}\pi\right)$, write Z in the form $a+ib$ where $a, b \in \mathbb{R}$ 3marks
- i) Let $Z_1 = 4+3i$ and $Z_2 = -2 +4i$. Using these values, show that $|z_1 + z_2| \leq |z_1| + |z_2|$ 4marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

- a) Use De-Moivre's theorem to show that $\sin 4\theta = 4 \cos^3 \theta \sin \theta - 4 \cos \theta \sin^3 \theta$ 5marks
- b) Using definition of exponentials, prove that $|e^{iz}| = e^{-y}$ 3marks
- c) Find the polar equation form of the Cartesian equation $(x^2+y^2)^2 + 8(x^2+y^2) = 4x^2$ 3marks
- d) Find the foci, vertices, eccentricity, length of latus rectum and the equation of the directrix for the conic $9x^2+90x+4y^2-16y+205 = 0$ 5marks
- e) Find the equation of the tangent to the parabola $y^2-4y-4x+8 = 0$ at point $P\left(\frac{5}{4}, 1\right)$ 4marks

QUESTION THREE (20 MARKS)

- a) Find all the values of Z such that $e^{2z-1} = 1$ 5marks
- b) Find the equation of a straight line passing through the intersection of the lines $2x+4y = 1$ and $2x+6y = 8$ and perpendicular to the $2x-4y+11 = 0$ 4marks
- c) Find the equation of an ellipse given its foci are $f_1(4,-2)$ and $f_2(4,6)$ and the length of its major axis is 16 units 4marks
- d) Sketch the curve of the equation $r = 6 \sin \theta$ 3marks
- e) Find the equation of the hyperbola through a vertex $V(5,-4)$ and with asymptotes $y + 4 = \pm \frac{3}{2}(x - 3)$ in the form $Ax^2 + By^2 + Cx + Dy + E = 0$ 4marks

QUESTION FOUR (20 MARKS)

- a) Find the intersection of circles $x^2+y^2+7x-8y-36 = 0$ and $x^2+y^2-24x+26y-312 = 0$ 7marks
- b) Let Z_1 and Z_2 be two complex numbers, then show that $|Z_1Z_2| = |Z_1||Z_2|$ 3marks
- c) Evaluate $\frac{(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3})^{11}}{(\cos\frac{4\pi}{3} + i \sin\frac{4\pi}{3})^7}$ 4marks
- d) Find the acute angle between the lines 6marks
- $L_1: x = -7t, y = 2+t, z = 3+2t$ and line
- $L_2 : x = 5-s, y = 3+2s, z = -2-6s$