MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

FIRST YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

AMC 102 – FINANCIAL MATHEMATICS I

DURATION: 2 HOURS

DATE: 16/04/2019

TIME: 2:00-4:00 PM

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Differentiate between interest rate and discount rate as used in finance 2marks

b) i) How many days does it take for £1450 to accumulate to £1500 if it is deposited in an account of paying 4% per annum simple interest? 2marks

ii) How long would it take if instead interest was 4% p.a. compound? 2marks

iii) Explain why the period under compound interest is different from that under simple interest.

2marks

c) i) Define the term annuity as used in financial mathematics 1marks

ii) Ken currently aged 20 years, plans to make contributions of Ksh. 5000 each year to an individual retirement arrangement starting this year. The account earns an interest of 4% per annum. He will not make a contribution at age 60, so that he will have made 40 payments in total. Calculate the sum of money he will have accumulated in his retirement fund at age 60 when he can withdraw funds without a penalty 4marks

d) A project has an initial outlay of Ksh 150,000. At the end of the first year and second year, there will be expenses of Ksh 20,000 and Ksh 15,000 respectively. The incomes from the project are Ksh 20,000 at the end of each of the first 4 years and Ksh 200,000 at the end of the 5^{th} year. Calculate the Internal Rate of Return (IRR) to 2 decimal places for the project 6 marks

e) For each of the following scenarios, compute the equivalent annual rate of interest

i) Nominal annual rate of discount is 7% p.a. convertible 3 times a year 2marks

ii) Nominal annual rate of discount is 0.5% p.a. compounded 6 times a year 2marks

iii) Nominal annual rate of interest is 10% p.a. convertible quarterly 2marks

f) A four year loan of £5000 is repaid by equal annual installments at the end of each year. Compute the annual payment on the basis of an interest rate of 6% p.a. effective; hence draw up

a loan schedule showing the interest and principal component of every payment and the outstanding balance. 5marks

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) The accumulated value just after the last payment under a 12-year annuity immediate of Ksh. 1000 per year, paying interest at the rate of 5% p.a. effective is used to purchase a perpetuity of Ksh. 500 per annum. The payments are to be made forever, with the first payment due 1 year after the last payment under the annuity. Showing all your work, determine the effective annual rate of interest for the perpetuity. 6marks

b) An investment fund is valued at Ksh 120 million on 1st January, 2010, and at Ksh 140 million on 1st January, 2011. Immediately after the valuation on 1st January 2011, Ksh 200 million is paid into the fund. On 1st January 2012, the value of the fund is Ksh 600million.

i) Calculate the annual effective time weighted rate of return over the two year period. 4marks

ii) Explain why the money-weighted rate of return would be higher than the time-weighted rate of return. 2marks

c) The nominal rate of discount is 8% per annum convertible quarterly. Calculate the equivalent:

i) rate of discount per annum effective 2marks

ii) rate of interest per annum effective 2marks

iii) rate of interest per annum convertible monthly 2marks

iv) rate of discount per annum convertible daily 2marks

QUESTION THREE (20 MARKS)

a) Differentiate between an annuity due and an annuity immediate giving a suitable example of each. 2marks

b) To accumulate Ksh 8000 at the end of 3*n* years, deposits of Ksh 98 are made at the end of each of the first *n* years and Ksh 196 an the end of the next 2*n* years. The annual effective rate of interest is *i*. Given that $(1+i)^n = 2$, determine the value of *i*. 5marks

c) The force of interest $\delta(t)$ is 0.05 for $0<t\leq10$; 0.006 for $10<t\leq20$ and $0.003t + 0.0002t^2$ for $t>20$. Calculate the:

i) Present value of a unit sum of money due at time t = 25 6marks

ii) Constant rate of interest per annum convertible monthly which gives the same result as in i) above.

iii) A Continuous payment stream is paid at the rate of $e^{-0.03t}$ per unit of time between time t=0 and t=5. Calculate the present value of that payment system. 3marks

QUESTION FOUR (20 MARKS)

a) You can receive either of the following two sets of cash flows:

Option A: Ksh. 5000 at the end of each year for the next 10 years.

Option B: Ksh X at the beginning of each year forever.

In both cases, the effective annual rate of interest is 10%. Find the value of X so that you are indifferent between these two options 4marks

b) A loan of £5000 is repaid by 15 annual payments of £500 each, with the first payment due in 1 year's time. Determine the annual effective rate of interest charged, using two methods;

i) The annuity formula 4marks

ii) The linear interpolation formula with trial values of i= 0.05 and i = 0.06 (Hint: The present value of an annuity immediate of amount 1 with term 15 years is 10.38 at i=5%, and 9.71 at i=6%) 4marks

c) The force of interest is a function of time and at any time t measured in years is given by the piecewise formula: $\delta(t)$=0.06 for $0 \leq t \leq 4$; 0.10-0.01t for $4 < t \leq 7$; and 0.01t-0.04 for t>7.

i) Calculate the value at time t=5 of £1000 due for payment at time t=10. 5marks

ii) Calculate the constant rate of interest per annum convertible monthly that is equivalent to the above force of interest. 3marks