MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

FOURTH YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA2405: COMPLEX ANALYSIS II

DURATION: 2 HOURS

DATE: 18TH APRIL 2018

TIME: 2.00 – 4.00PM

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

a) Locate and name all the singularities of the function $f(z) = \frac{z^4+1}{z^2(z-1)}$ (3 Marks)

b) Use the expansion method to find the residue of the function $g(z) = ze^{5/z}$ (3 Marks)

c) Evaluate

$$\oint \frac{1-2z}{z(z-1)(z-3)}$$

Where C is the circle $|z| = 2$ (5 Marks)

d) Use Cauchy's residue theorem to evaluate

$$\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$$

(5 Marks)

e) Show that if the function $f(z)$ is analytic and $f'(z)$ is not equal to zero in a region R, then the mapping $w = f(z)$ is conformal at all points of the region R. (5 Marks)

f) Show the function $u = xe^x \cos y - ye^x \sin y$ (4 Marks)

g) Show that the functions

$$g(z) = \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$$

And

$$f(z) = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(2-i)^{n+1}}$$

Are analytic continuations of each other. (5 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Compute the residue at the singularities of the function $g(z) = \frac{\cos z}{z^2(z-\pi)^3}$ (9 Marks)

b) Evaluate $\oint_C \frac{e^{zt}\, d^z}{z^2(z^2+2z+2)}$ where C is the circle $|z| = 3$. (11 Marks)

QUESTION THREE (20 MARKS)

a) Prove that $u = 3x^2y + 2x^2 - 2y^2$ is harmonic and find a function V such that $f(z) = u + iv$ is analytic. Express $f(z)$ in terms of z. (10 Marks)

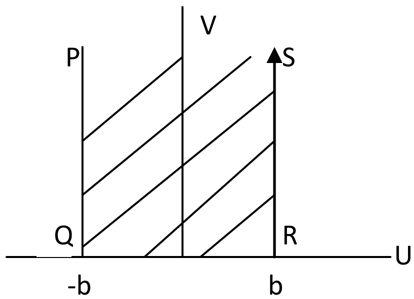
b) Show that the composition of two bilinear complex mappings is also a bilinear mapping.

(10 Marks)

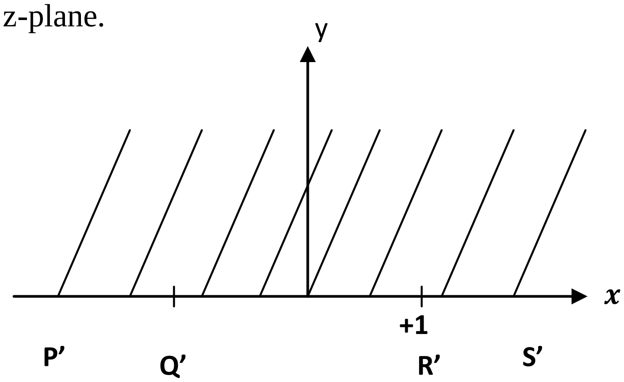
QUESTION FOUR (20 MARKS)

a) Find the image of a circle $|z - i| = 1$ under the transformation $w = \frac{z-i}{z+i}$ (10 Marks)

b) Use the Schwarz-Christofell transformation to determine a function which maps the region in the w-plane shown onto the upper half of the z-plane.



w-plane



z-plane

(10 Marks)