MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

RE-SIT/SPECIAL EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA2202 – ALGEBRAIC STRUCTURES

DURATION: 2 HOURS

DATE: 24TH APRIL 2018

TIME: 2.00 – 4.00P.M

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

a) Define the following terms;

i. A mapping (2 Marks)

ii. Binary Operation (2 Marks)

iii. Semi-group (2 Marks)

b) Consider $< S, X >$ where $S = \{1, -1, i, -i\}$. Draw the Cahley table for the system. (5 Marks)

c) Determine the order of the elements in 1(b) above. (2 Marks)

d) Let $*$ be a binary operation on $\mathbb{R}$ by $a * b = \frac{ab}{a+b}$. is $*$an operation on $\mathbb{R}$. (3 Marks)

e) Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2X + 3$ and $g(x) = 3X + 2$. Compute;

i. $f \circ g)(2)$ (2 Marks)

ii. $F \circ g \supset g \circ f \circ g$ (3 Marks)

iii. $(g \circ f)^{-1}$ (3 Marks)

f) State and prove the Latin square property of a finite group. (6 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Define the following terms;

i. Cyclic group (2 Marks)

ii. Dihedral group (2 Marks)

iii. Subgroup (2 Marks)

b) Prove the union of two subgroups is not necessary a subgroup of a group G. (5 Mark)

c) Let $G = \mathbb{Z}_6$ and $H = \{0,2,4\}$. Determine the left cosets of H in G. (5 Marks)

d) Let $< G, \ *>$ be a group with the identity e_1. Show that this identity is unique. (4 Marks)

QUESTION THREE (20 MARKS)

a) Define the term commutative ring and give an example of a commutative ring. (4 Marks)

b) Compute all zero divisors of $\mathbb{Z}_8$ (4 Marks)

c) Let R be a ring and $a \in R$ be a nilpotent element in R. Prove that a is a zero divisor. (6 Marks)

d) Show that $R =< \mathbb{Z}_n, +, X >$ is an integral domain. (4 Marks)

e) Define the term unit in a ring. (2 Marks)

QUESTION FOUR (20 MARKS)

Let $S = \{(x, y)|x, y \in \mathbb{R}\}$ and $*$ on S be defined as $(a, b) * (c, d) = (a + bc, bd)$. Determine the following;

i. Is S closed under $*$? (2 Marks)

ii. Is $*$ commutative on S? (3 Marks)

iii. Is $*$ associative on S? (3 Marks)

iv. Does $*$ admit an identity in S? (5 Marks)

v. Does $*$ admit inverse in S? (5 Marks)

vi. Is $< S,* >$ a group? (2 Marks)

QUESTION FIVE (20 MARKS)

a) Give two applications of algebraic structures in computer science. (2 Marks)

b) Define the term field and give examples of fields. (5 Marks)

c) Let $T = \left\{\begin{bmatrix} t & 0 \\ 0 & 0 \end{bmatrix}, t \in \mathbb{R}^*\right\}$ multiplication is a binary operation on T. Check:

i. Whether multiplication admits an identity in T. (6 Marks)

ii. What is the inverse of an element B in T. (7 Marks)