MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

FOURTH YEAR **SECOND** SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE

SMA 2437 – MULTI VARIATE METHODS

DURATION: 2 HOURS

DATE: 18TH APRIL, 2018

TIME: 9.00 – 11.00 A.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

a) Define the following as used in multivariate methods

i. Multivariate analysis (2 marks)

ii. Multivariate data (2 marks)

b) Given that $\underline{X} \sim N_3\ (\mu, \Sigma)$ with $\Sigma = \begin{pmatrix} 121 & -27.5 & 15.4 \\ -27.5 & 25 & 7 \\ 15.4 & 7 & 49 \end{pmatrix}$ Find the correlation matrix for the data hence the partial correlation between x_3 and x_2 for fixed level of x_1 (6 marks)

c) Given that A = $\begin{pmatrix} 36 & -\frac{18}{5} & 0 \\ -\frac{18}{5} & 9 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Find the eigen values of the matrix and show that their product equals its determinant while their sum equals its trace (6 marks)

d) Let $X \sim N(\mu, \varepsilon)$ be trivariate normal random vector with $\mu = \begin{pmatrix} 35 \\ 28 \\ 14 \end{pmatrix}, \Sigma = \begin{pmatrix} 36 & 18 & \frac{42}{5} \\ 18 & 81 & -\frac{63}{2} \\ \frac{42}{5} & -\frac{63}{2} & 49 \end{pmatrix}$

Find:

i. The regression function of X_1 on x_2 and x_3 (4 marks)

ii. The conditional variance of X_1 when given x_2 and x_3 (2 marks)

e) Suppose the mean vector and variance - covariance matrix of $\underline{X} = [x_1, x_2, x_3, x_4]$ given as

$\underline{\mu} = \begin{bmatrix} 2 \\ 6 \\ 5 \\ 3 \end{bmatrix}$ and variance covariance matrix is given as $\Sigma = \begin{bmatrix} 9 & 4 & -1 & 2 \\ 4 & 5 & 1 & 3 \\ -1 & 1 & 7 & -4 \\ 2 & 3 & -4 & 6 \end{bmatrix}$

i. Find the variance – covariance matrix of $[x_2,\ x_4]$ and $[x_1,\ x_3]$ (3 marks)

ii. Total and generalized variance of $x_1,\ x_3$ (2 marks)

f) Give an expression for multivariate normal distribution and explain the meaning of non-singular multivariate normal distribution (3 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

a) Given that $\underline{x} \sim N_3\ (\mu, \varepsilon)$ with $\mu = \begin{pmatrix} 20 \\ 45 \\ 46 \end{pmatrix}, \Sigma = \begin{pmatrix} 36 & 12 & -6 \\ 12 & 16 & 2 \\ -6 & 2 & 4 \end{pmatrix}$

Find:

i. The distribution of $y = \begin{pmatrix} x_1 - & 2x_2 & \\ 2x_1 + & x_2 - & x_3 \end{pmatrix}$ (6 marks)

ii. The distribution of x_1 given that $x_2 = 40$ and $x_3 = 48$ (4 marks)

iii. The regression function of x_3 on x_1 and x_2 (4 marks)

iv. The partial correlation coefficient between x_1 and x_2 for fixed values of x_3 and that of x_1 and x_3 for fixed values of x_2 (6 marks)

QUESTION THREE

a) State the assumptions behind the idea of using MANOVA (4 marks)

b) Observations on three responses are collected for two treatments as shown in the table below:

Treatment	1	1	1	1	1	2	2	2
x_1	11	12	8	7	7	11	12	10
x_2	18	17	15	12	13	11	13	15
x_3	7	8	7	7	6	14	13	12

Find:

i. The matrix of sum of squares due to treatment (6 marks)

ii. The matrix of residual sum of squares (6 marks)

iii. The Wilk's lambda statistic and use it to test the hypothesis that there is no treatment effects (use $\propto = 0.05$) (4 marks)

QUESTION FOUR

a) State and explain any three objectives of scientific observations based on multivariate data (6 marks)

b) Some students did a test in three units and the results were as follows:

Test 1	9	2	6	5	8
Test 2	12	8	6	4	10
Test 3	3	4	0	2	1

Determine:

i. The mean vector (2 marks)

ii. Variance – covariance matrix (5 marks)

iii. Sample correlation matrix and interpret its elements (7 marks)

QUESTION FIVE

a) Let $\underline{x} \sim N_3\ (\mu, \varepsilon)$ be trivariate normal random vector. Suppose a certain sample

$$S = \begin{pmatrix} 16 & 0 & 0 \\ 0 & 25 & \frac{5}{2} \\ 0 & \frac{5}{2} & 4 \end{pmatrix}$$ Find:

i. The eigen values of this matrix (7 marks)

ii. The first principal components (4 marks)

iii. The total variance explained by the component (4 marks)

b) Consider the following two normal populations

$\pi_1\ N_p(\mu_1, \Sigma)$

$\pi_2\ N_p(\mu_2, \Sigma)$

Suppose an observation x is to be classified into one of the populations. Derive a discriminant rule for classifying x (5 marks)