MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

FOURTH YEAR **SECOND** SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF MATHEMATICS & COMPUTER SCIENCE

SMA 2421 – NUMERICAL ANALYSIS II

DURATION: 2 HOURS

DATE: 27TH APRIL, 2018

TIME: 9.00 – 11.00 A.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

a) When is a polynomial P(x) referred to as an interpolating polynomial? (1 mark)

b) Give the Taylor series expansion for the function f(x) about a point $x_o, x_0 \in [a, b]$ (2 marks)

c) Obtain the Taylor series approximation about x=1, upto second degree terms for the function f(x) = $\frac{1}{1+x^2}$, and find a bound on the error if this approximation is to be used in [1, 1.4] (7 marks)

d) Obtain a linear polynomial approximation to the function f(x) = x^2 on the interval [0,2] using the least squares approximation with w(x) = 1 (6 marks)

e) Prove that the legendre polynomial of order four is given by

$P_4(x) = \frac{35}{8}x^4 - \frac{15}{4}x^2 + \frac{3}{8}$ (6 marks)

f) Give the mathematical expression of the following:

i. Eigen value problem (1 mark)

ii. Characteristic equation (1 mark)

iii. Characteristic vector (1 mark)

g) Consider the 2 x 2 matrix A = $\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$ identify the eigen values and hence the eigen vectors (5 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

a) Solve the system of equations:

$10x_1 - x_2 + 2x_3 = 4$

$x_1 + 10x_2 - x_3 = 3$

$2x_1 + 3x_2 + 20x_3 = 7$

Using the Gauss elimination method, apply pivoting where necessary (7 marks)

b) Apply the lagrange formula to find the unique polynomial of degree 2 which fits the given data.

x	0	2	4
f(x)	1	12	18

(6 marks)

c) Find the approximation to the initial value problem $y' = -y + 1 \;\; 0 \le t \le 1$

$y(0) = 0 \;\; N = 10$ & $t_i = a + {}_i h \;\; h = 0.025$ for w_1, w_2, w_3 using Euler's method (4 marks)

d) Apart from the Euler's method, give any three other methods that can be applicable in determining the numerical solution y(t) of the problem

$\frac{dy}{dx} = f(t, y)$for $a \leq t \leq b$ (3 marks)

QUESTION THREE

a) Solve the system of equations

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 5 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ -4 \end{bmatrix}$$

Using the Jacobi iteration method

Take the initial approximations as $x^{(0)} = [0, 0, 0]^T$ and perform three iterations in each case. The exact solution is $x_1 = 1, x_2 = -1$ and $x_3 = -1$ (10 marks)

b) i. In maximization, give the meaning of the following:

- Pivot number (1 mark)
- Basic solution (1 mark)
- Basic variables (1 mark)
- Basic feasible solution (1 mark)

ii. Use simplex method to find the maximum value of P = x + 4y subject to;

$-x + 2y \leq 6$

$5x + 4y \leq 40$

$x, y \geq 0$

Perform two iterations only (6 marks)

QUESTION FOUR

a) Given the following values of f(x) and f '(x)

x	f(x)	f '(x)
-1	1	-5
0	1	1
1	3	7

Estimate the values of f(-0.5) and f(0.5) using the Hermite interpolation. The exact values are;

f(-0.5) = $\frac{3}{8}$ and f(0.5) = $\frac{11}{8}$ (8 marks)

b) The following data for the function $f(x) = x^4$ is given;

x	0.4	0.6	0.8
f(x)	0.0256	0.1296	0.4096

Find f '(0.8) and f"(0.8) using quadratic interpolation. Compare with the exact solution

(4 marks)

c) Solve the system of equations;

$2x_1 - x_2 + 0x_3 = 7$

$-x_1 + 2x_2 - x_3 = 1$

$0x_1 - x_2 + 2x_3 = 1$

Using the Gauss seidel method. Take the initial approximation as $x^{(0)} = 0$ and perform three iterations. Exact solution is $x = [6, 5, 3]^T$ (8 marks)

QUESTION FIVE

a) i. Prove that the Chebyshev polynomial of order four is given by $T_4(x) = 8x^4 - 8x^2 + 1$

(5 marks)

ii. By taking m=n=2 show that $P_n(x)$ as orthogonal polynomial satisfy

$\int_{-1}^{1} P_m(x)P_n(x)dx = \frac{2}{2n+1}$ (5 marks)

b) By applying the Gauss quadrature methods, derive;

i. One point formulae for Gauss legendre integration method (2 marks)

ii. Evaluate the integral

$I = \int_1^2 \frac{2xdx}{1+x^4}$ using the Gauss-legendre

1 – point quadrature rule (4 marks)

c) i. Define Matlab (1 mark)

ii. Give three basic operations that can be done by Matlab (3 marks)