



# MURANG'A UNIVERSITY OF TECHNOLOGY

## SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**FOURTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF  
BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE**

SMA 2402 – TOPOLOGY II

DURATION: 2 HOURS

DATE: 20<sup>TH</sup> APRIL, 2018

TIME: 2.00 – 4.00 P.M.

### **Instructions to Candidates:**

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

## SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

### QUESTION ONE

- a) i. What is a regular space (2 marks)  
ii. Show that a regular space need not be a T space (4 marks)
- b) State without prove Lindenof's theorem (2 marks)
- c) Differentiate between a first countable space and second countable space (2 marks)
- d) Let  $A$  be a compact subset of a compact subset of a Hausdorff space  $X$  and suppose  $P \in X/A$ , show that there exists open sets  $G$  and  $H$  such that  $P \in G, A \subset H$  and  $G \cap H = \emptyset$  (5 marks)
- e) Show that an open interval  $A=(0,1)$  on the real line  $\mathbb{R}$  with the usual topology is not sequentially compact (2 marks)
- f) Prove that every bounded closed interval  $B=[a, b]$  is countably compact (3 marks)
- g) Consider a topology on  $X=\{a,b,c,d,e\}$  defined by  $\tau = \{X, \emptyset, \{a, b, c\}, \{c, d, e\}, \{c\}\}$  prove that  $A= \{a, d, e\}$  is disconnected (3 marks)
- h) Let  $F: I \rightarrow X$  be a path from  $a$  to  $b$  and Let  $g: I \rightarrow X$  be a path from  $b$  to  $c$ . show that the juxtaposition of the two paths  $F$  and  $g$  denoted by  $F * g$  is the function  $F * g: I \rightarrow X$  defined by
- $$(F * g)(s) = \begin{cases} F(2s) & 0 \leq s \leq 1/2 \\ g(2s - 1) & 1/2 \leq s < 1 \end{cases} \quad (4 \text{ marks})$$
- i) Prove that the component of a totally disconnected space  $X$  are singleton subsets of  $X$  (3 marks)

## SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

### QUESTION TWO

- a) Define a sequentially compact subset  $A$  of a topological space  $X$  (2 marks)
- b) Show that if  $A$  is a finite subset of a topological space  $X$ , then  $A$  is sequentially compact (3 marks)
- c) Let  $A$  be a subset of topological space  $X$ . Prove that if  $X$  is sequentially compact then  $A$  is countably compact (5 marks)
- d) Differentiate between the terms separated sets and a disconnected set (4 marks)
- e) Let  $G \cup H$  be a disconnection of  $B$ . show that  $B \cap G$  and  $B \cap H$  are separated sets (6 marks)

### QUESTION THREE

- a) i. Define what is meant by one-point compactification (2 marks)  
ii. Prove that a one point compactification  $\tilde{X}$  of a  $T_1$ -space  $X$  is a compact  $T_1$ -space (5 marks)
- b) Let  $X$  and  $Y$  be topological spaces and  $F: X \rightarrow Y$  continuous and onto function. Prove that if  $X$  is connected then  $Y$  is also connected (4 marks)
- c) Prove that every projection  $\pi_i: X \rightarrow X_i$  on a product space  $X = \prod X_i$  is both open and continuous (5 marks)
- d) Show that any infinite subset of a discrete topological space  $X$  is not compact (4 marks)

### QUESTION FOUR

- a) Prove that every archwise connected set  $A$  is connected (5 marks)
- b) Explain what is meant by a subset of a topological space  $X$  is compact and hence show that every compact subset of a Hausdorff space  $X$  is closed (5 marks)
- c) Show that a continuous image of a compact set is also compact (6 marks)
- d) Explain what is meant by class  $\{B_i\}$  of sets is said to have a finite intersection property and hence determine if the class  $\beta_i = \{(0,1), (0, \frac{1}{2}), (0, \frac{1}{3}), \dots\}$  of interval in  $\mathbb{R}$  has the finite intersection property (4 marks)

### QUESTION FIVE

- a) Let  $\mathcal{G}$  be a base for a second countable space  $X$ . Then prove that  $\mathcal{G}$  is reducible to a countable base for  $X$  (5 marks)
- b) Let  $A$  be a subset of a topological space  $(X, \tau)$  and Let  $\tau_A$  be the relative topology of  $A$ , then show that  $A$  is connected with respect to  $\tau$  if and only if  $A$  is connected with respect to  $\tau_A$  (5 marks)
- c) Consider the topology  $\tau = \{X, \emptyset, \{1\}, \{2,3\}\}$  on  $X = \{1,2,3\}$  and the topology  $\tau^* = \{Y, \emptyset, \{a\}\}$  on  $Y = \{a,b\}$ .  
i. Determine the defining subbase  $\delta$  of product topology on  $X \times Y$  (5 marks)  
ii. Determine the defining base  $\beta$  for the product topology on  $X \times Y$  (2 marks)
- d) Let  $A = (0,1)$  and  $B = (1,2)$ . Show that  $A$  and  $B$  are disconnected (3 marks)