MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

~~**FOURTH**~~ YEAR , ~~**SECOND**~~ SEMESTER EXAMINATION FOR ~~BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE~~

AMS301: THEORY OF ESTIMATION

DURATION: 2 HOURS

DATE: 26TH APRIL 2018

TIME: 2.00PM – 4.00PM

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

a) Define the following terms (3 Marks)
   i. Loss function
   ii. Consistent estimator
   iii. Unbiasedness

b) Let $f(x,u) = \begin{cases} \frac{2}{u^2}(u-x) \end{cases}$ for $0 < x < u \; and \; 0 < u < \infty$. Let $x_1, x_2 \cdots x_n$ be a random sample from X obtain the estimate of u by method of moments. (5 Marks)

c) If X is a Poisson random variable with parameter λ. Find the maximum likelihood estimator of λ. (5 Marks)

d) Briefly explain four properties a good estimator should possess. (4 Marks)

e) Let $x_1, x_2 \cdots x_n$ be a random sample from a population given by

$$f(x;u) = \begin{cases} 1 & for\; u - {}^1/_2 < x < u + {}^1/_2 \\ 0 & elsewhere \end{cases}$$

Prove that $\bar{x}$ is a consistent estimator for u. (5 Marks)

f) Briefly explain the condition necessary for finding a sufficient statistic using factorization criteria. (4 Marks)

g) Suppose $x_1, x_2 \cdots x_n$ forms a random sample from a Bernoulli distribution for which probability of success u is unknown $0 < u < 1$. Show that $T = \varepsilon x_i$ is a sufficient statistic for u. (4 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Show that the family of Bernoulli distribution belongs to one parameter exponential family. Hence or otherwise obtain a sufficient statistic for $\theta[f(x;\theta) = \theta^x(1-\theta)^{1-x}]$. (10 Marks)

b) Show that $S^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$ is a biased estimator of δ^2 while $S^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$is unbiased. (10 Marks)

QUESTION THREE (20 MARKS)

a) In an exponential population where the probability density function is given by

$$f(x;u) = \begin{cases} \frac{1}{4} e^{-x/u}; x > 0 \\ 0 \quad elsewhere \end{cases}$$

Show that $\bar{x}$ is an unbiased estimator for u. (10 Marks)

b) Obtain by method of moments estimator for u in an exponential population. Identity for exponential is

$$f(x;u) = \begin{cases} ue^{-ux} & \begin{matrix} 0 \leq x < 0 \\ u > 0 \end{matrix} \\ 0 & elsewhere \end{cases}$$

(10 Marks)

QUESTION FOUR (20 MARKS)

a) State the Cramer-Rao theorem for finding the lower bound of the variance of an unbiased estimator. (7 Marks)

b) Let X be a Poisson random variable with parameter $\lambda > 0$. Let $x_1, x_2 \cdots x_n$ be a random sample provided. Find the uniformly minimum variance unbiased estimator of λ.

(10 Marks)

c) State any three properties of a good estimator. (3 Marks)