



MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**SECOND YEAR , ~~SECOND~~ SEMESTER EXAMINATION FOR BACHELOR
OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING**

AMS208: PROBABILITY AND DISTRIBUTION THEORY II

DURATION: 2 HOURS

DATE: 18TH APRIL 2018

TIME: 2.00PM – 4.00PM

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

- a) Differentiate between a parameter and a statistic as used in sampling theory. (2 Marks)
- b) Let x_1, x_2, \dots, x_n be random variables each with mean μ and standard deviation $\sigma > 0$. Assume that each pair of x_i, x_j of random variables with $i \neq j$ is uncorrelated. Let

$$S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{X})^2$$

be the sample variance. Show that $E(S^2) \neq \sigma^2$ (5 Marks)

- c) A random sample of size 10 is taken from a normally distributed population with unknown variance. The sample mean \bar{X} and standard deviations are given by 38.4 and 5.5. Calculate the 98% confidence interval for the variance (σ^2). (4 Marks)
- d) Let x_1, x_2, \dots, x_n be a random sample size n taken from a probability distribution where S^2 is the population variance ($\sigma^2 > 0$). Show that $E(S^2) = \frac{n-1}{n} \sigma^2$. (4 Marks)
- e) State and prove the weak law of large numbers. (5 Marks)
- f) Let x_1, x_2, \dots, x_n be a sample from a normal population with mean μ and variance σ^2 . Show that the variable $\frac{n-1}{\sigma^2} S^2$ has a chi-square distribution with $n - 1$ degrees of freedom. (5 Marks)

- g) Let X be a continuous random variable with the following p.d.f.

$$f(x) = \begin{cases} 5x^4 & \text{for } 0 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

- i. Find the probability density function of $Y = X^3$ (3 Marks)
- ii. Determine $P(\frac{1}{3} < y < 2)$ (2 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

- a) Let x_1, x_2, x_3, x_4 be four independent sample observations of Poisson distribution with parameter θ . Show that

$$T = \frac{3x_1 + 2x_2 + 3x_3 + 2x_4}{10}$$

is unbiased estimator of θ . (3 Marks)

- b) The joint density of X_1 and X_2 is given by

$$f(x_1, x_2) = \begin{cases} 6e^{-3x_1-2x_2}, & 0 < x_1 < x_2, 0 < x_2 < \infty \\ 0 & \text{Otherwise} \end{cases}$$

Find the density function of Y where $Y = X_1 + X_2$ by distribution function technique.

(6 Marks)

- c) A population consists of the five numbers 2,3,6,8 and 11. Consider all the possible samples of size two which can be drawn with replacement from this population. Find;

- i. The mean of the population. (1 Mark)
- ii. The standard deviation of the population. (3 Marks)
- iii. The mean of the sampling distribution of the mean. (4 Marks)
- iv. The standard deviation of the sampling distribution of the means. (3 Marks)

QUESTION THREE (20 MARKS)

- a) State and prove the central limit theorem for independent and identically distributed random variables. (9 Marks)
- b) The time taken to complete a certain test is assumed to be normally distributed with a mean μ and variance σ^2 . A random sample of size 15 is taken and the following observed. 119.2, 128.9, 112.8, 120.9, 91.1, 115.3, 111.9, 92.0, 181.5, 123.6, 146.2, 99.0, 131.7, 141.7 and 114.9. Find the 90% confidence interval for the variance. (5 Marks)
- c) A random sample of 16 value form a normal population showed a mean of 41.5cm and the sum of squares of the deviations from this mean equal to 125cm². Obtain the 95% and 99% confidence limits for the population mean. (6 Marks)

QUESTION FOUR (20 MARKS)

- a) Let x_1, x_2, \dots, x_n be random variables distributed normally and independently with mean μ_i and variance (σ^2) where $i = 1, \dots, n$. Show that

$$U = \sum_{i=1}^n \left(\frac{x_i - \mu_i}{\sigma} \right)^2$$

has a Chi-square distribution with n degrees of freedom. (6 Marks)

- b) Let U_1 and U_2 be independent random variables where U_1 has a Chi-square distribution with $\sqrt{2}$ degrees of freedom. Derive the probability distribution of the random variable

$$F = \frac{U_1}{\sqrt{1}} / \frac{U_2}{\sqrt{2}}$$

(14 Marks)