MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

SECOND YEAR **SECOND** SEMESTER EXAMINATION FOR B.Sc. IN APPLIED STATISTICS WITH PROGRAMMING, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE AND BACHELOR OF SCIENCE IN MATHEMATICS AND ECONOMICS

AMS202 – PROBABILITY AND STATISTICS II

DURATION: 2 HOURS

DATE:

TIME:

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

a) Distinguish between continuous and discrete random variables. (2 Marks)

b) A continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} 12x^2(1-x) & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

Evaluate;

i. $E(x)$ (3 Marks)

ii. $Var(x)$ (3 Marks)

iii. $E[x^2 + 3x + 4]$ (3 Marks)

c) The probability distribution function $P(x)$ of a discrete random variable X is given by

$$P(x) = \begin{cases} Kx(x+2) & s = 1,2,3,4 \\ 0 & Otherwise \end{cases}$$

i. Determine the value of K for $P(x)$ to the valid PMF. (3 Marks)

ii. Evaluate the following probabilities

$P(1 < X \leq 3$ (2 Marks)

$P(X \leq 2)$ (2 Marks)

d) In a sample of 1000 students the mean of a certain test is 14 and the variance is 6.25. Assuming the distribution to be normal, find

i. How many students scored between 12 and 15? (2 Marks)

ii. How many scored above 18? (2 Marks)

e) The moment generating function $M_{x(t)}$ of a random variable X is given by

$$M_{x(t)} = \left({}^{3}/_{5} + {}^{2}/_{5}\, e^{t}\right)^{10}$$

Determine;

i. Mean of x (3 Marks)

ii. Variance of x (4 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Suppose that X has a gamma distribution with parameter $\propto = 2\ and\ \beta = {}^{1}/_{4}$. Compute $P(4.32 < X < 36.6)$ (8 Marks)

b) Let X be a random variable with density function $f(x)$ given as

$$f(x) = \begin{cases} 6x(1-x) & 0 < x < 1 \\ 0 & Otherwise \end{cases}$$

Find the 1st three raw moments. Hence determine the mean and variance of x. (6 Marks)

c) It is known that in a batch of 100 transistors, 20 of them are defective. 10 are selected for inspection. What is the probability that;

i. All 10 are defective. (2 Marks)

ii. At least 2 are defective. (2 Marks)

iii. At most 3 are defective. (2 Marks)

QUESTION THREE (20 MARKS)

a) The number of accidents per week in a certain factory follows a Poisson distribution with variance 3.2. Find the probability that

i. At least 6 but not more than 9 accidents occur per week. (3 Marks)

ii. At most 5 accidents occur in a particular fortnight. (4 Marks)

b) X is a continuous random variable with a probability function given by

$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

i. Determine the cumulative distribution of x (3 Marks)

ii. Calculate the median and the quartile deviation of x. (6 Marks)

c) A random variable X has a Poisson distributin such that

$$P(X = 2) = {}^{2}/_{3}\, P(X = 1)$$

Evaluate $P(X = 0)$ (4 Marks)

QUESTION FOUR (20 MARKS)

a) Find the $m.g.f.$ of the distribution below and use it to find the mean and variance of x

$$P(X = x) = \begin{cases} \binom{n}{x} p^x q^{n-x} & x = 0,1,2 \cdots n \\ 0 & otherwise \end{cases}$$

(8 Marks)

b) A discrete random variable has the probability function

x	0	1	2	3
$p(x)$	${}^{84}/_{286}$	${}^{144}/_{286}$	${}^{54}/_{286}$	${}^{4}/_{286}$

Calculate

i. The mean of x (2 Marks)

ii. The variance of x (2 Marks)

c) i) If $h(x: n, m, N)$ is the probability density function for hyper geometric distribution, show that

$$h(x+1, n, m, N) = \left[\frac{(n-x)(m-x)}{(x+1)(N-m-n+x+1)}\right] h(x; n, m, N)$$

(5 Marks)

ii) A shipment of 220 items contains 15 items that are defective. If 5 of these items are randomly selected and shipped to a customer, find the probability that the customer will get one bad item. (3 Marks)

QUESTION FIVE (20 MARKS)

a) A committee of 4 people is to be selected at random from among 10 people of whom 6 are female and 4 are men. If the desired outcome is male, determine;

i. The required distribution function. (2 Marks)

ii. The mean (2 Marks)

iii. The Variance. (2 Marks)

b) If on average rain falls on 10 days in every 30. Find the probability that the rain falls on at most 2 days of a given week. (6 Marks)

c) A continuous random variable y has a pdf

$$f(y) = \begin{cases} m + ny^2 & 0 < y < 1 \\ 0 & elsewhere \end{cases}$$

If its mean is $^2/_3$ determine the values of m and n. Hence calculate the variance of y.

(8 Marks)