MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

SECOND YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

AMS331: PROBABILITY AND STATISTICS IV

DURATION: 2 HOURS

DATE: 19TH APRIL 2018

TIME: 9.00AM – 11.00AM

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

a) Define a multivariate normal distribution explaining clearly all the parameters used. (3 Marks)

b) State three (3) properties of a multivariate joint density function. (3 Marks)

c) Suppose that X_1, X_2 are independent with density functions $f_1(X_1)$ and $f_2(X_2)$. Find the distribution of $U_1 = X_1 + X_2$ and $U_2 = X_1 - X_2$. (8 Marks)

d) State and proof the weak law of large numbers. (5 Marks)

e) A survey of 1500 people is conducted to determine whether they prefer Pepsi of Coke. The result shows that 27% of people prefer coke while the remaining 73% favour Pepsi. Estimate the margin of error in the poll with a confidence of 90%. (5 Marks)

f) Let the probability generating function of X be given by

$$G_X(S) = \frac{(1+s)^2 + (1+3s)}{16}$$

Find the mean and variance X. (6 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Suppose someone gives you a coin and claims that this coin is biased that it lands on heads only 48% of the time. You decide to toss the coin for yourself. If you want to be 95% confident that this coin is indeed biased, how many times must you flip the coin using;

i. The weak law of large numbers. (5 Marks)

ii. The central limit theorem. (5 Marks)

iii. Which method would you oft for? Explain. (2 Marks)

b) i. Derive the probability generating functions of a binomial distribution. (4 Marks)

ii. Use the probability generating function to obtain the mean and variance of the binomial. (4 Marks)

QUESTION THREE (20 MARKS)

a) Let X, Y and Z denote three jointly distributed random variables with joint density function

$$f(X,Y,Z) = \begin{cases} k(X^2 + Y^Z) & 0 \le X, Y, Z < 1 \\ 0 & Otherwise \end{cases}$$

i. Find the value of k (5 Marks)

ii. Calculate$E[xyz]$ (5 Marks)

iii. The marginal distribution of xy (4 Marks)

iv. The conditional distribution of z given $X = x, Y = y$ (3 Marks)

QUESTION FOUR (20 MARKS)

a) State any four properties of the covariance matrix. (4 Marks)

b) Suppose $X_1, \cdots, X_q$ are independent of $X_{q+1}, \cdots, X_k$, show that

$$E\left[g\left(X_1, \cdots, X_q\right) h\left(X_{q+1}, \cdots, X_k\right)\right] = E\left[g\left(X_1, \cdots, X_q\right)\right] E\left[h\left(X_{q+1}, \cdots, X_k\right)\right]$$

(6 Marks)

c) Suppose that we observe an experiment that has K possible outcomes $\{O_1, O_2, \cdots, O_k\}$ independent times let $P_1, P_2, \cdots, P_k$ denote probability of $O_1, O_2, \cdots O_k$ respectively. Let X_i denote the number of times that outcome O_i occurs in the n repetition of the experiment. Find the distribution of X_i. (4 Marks)

d) Suppose $X_1, \cdots, X_q$ and independent of $X_{q+1}, \cdots, X_k$, show that

$$E\left[g\left(X_1, \cdots, X_q\right) h\left(X_{q+1}, \cdots, X_k\right)\right] = E\left[g\left(X_1, \cdots, X_q\right)\right] E\left[h\left(X_{q+1}, \cdots, X_k\right)\right]$$

(6 Marks)