MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

THIRD YEAR, **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING AND BACHELOR OF SCIENCE IN MATHEMATICS AND ECONOMICS

AMS 313: TEST OF HYPOTHESIS

DURATION: 2 HOURS

DATE: 17TH APRIL 2018

TIME: 2.00PM – 4.00PM

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

a) Define the following terms as use in test of hypotheses;

  i. Statistical hypothesis (1 Mark)

  ii. Type I and Type II errors (2 Marks)

b) It is claimed that the mean weight of workers in a research station is 62kgs with a standard deviation of $\sigma = 12$. To test this claim a random sample of 225 workers was taken and $\bar{X} = 56$ was obtained. Does this provide sufficient evidence to reject the claim at $\propto = 0.01$ (4 Marks)

c) Let $x_1, x_2 \cdots x_n$ denote an independent random sample from a population with a Poisson distribution with mean λ. Derive the most powerful test for testing $H_0: \lambda = 2$ against $H_0: \lambda = {}^1\!/_2$. (4 Marks)

d) State the Neyman Pearson Lemma giving precisely what kind of test it is applied on. (4 Marks)

e) Let $x_1, x_2 \cdots x_n$ be a random sample from a probability distribution given by

$$f(x, \theta) = \begin{cases} \theta e^{-\theta x} \quad , & \theta > 0 \\ 0 \quad , & otherwise \end{cases}$$

Where $\theta = \theta_0$ or $\theta = \theta_1$ where θ_0 and θ_1 are constants. Derive the most powerful test for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ using Neyman-Pearson Lemma. (5 Marks)

f) Describe the sequential probability ratio test for testing of hypotheses. (5 Marks)

g) Mendelian theory indicates that the space and colour of a certain variety of bean can be grouped into 5 groups, round and yellow, round and green, angular and yellow, angular and green, and angular and white. According to the ratio 7:3:5:4:1, the following was observed.

Round and yellow	195
Round and green	200
Angular and Yellow	110
Angular and Green	120
Round and white	35

Is the Mendelian theory correct in this case at 5% level of significant? (4 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Let $x_1, x_2 \cdots x_n$ be a random sample from a Bernoulli distribution given by

$$f(x) = \begin{cases} p^x (1-p)^{1-x} & , x = 0,1 \\ 0 \quad , & Otherwise \end{cases}$$

By taking a random sample of size n derive a most power size$\propto$ test for testing $H_0: p = p_0$ against $H_1: p \neq p_1$ where $p_1 > p_0$ (4 Marks)

b) Let $x_1, x_2 \cdots x_n$ be a random sample from a normal population with mean μ and variance δ^2. Using the likelihood ration test, find the uniformly most powerful test of $H_0: \mu \leq \mu_0$ against $H_1: \mu > \mu_0$ by considering the simple hypotheses $H_0: \mu = \mu_0$ against $H_1: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$ when

i. $\mu_1 > \mu_0$ with known δ^2 (4 Marks)

ii. δ^2 is unknown. (12 Marks)

QUESTION THREE (20 MARKS)

a) Briefly outline the generalized likelihood ratio test. (3 Marks)

b) A single observation is drawn from the distribution

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}; x \geq 0, \delta > 0 \\ 0 \quad elsewhere \end{cases}$$

It is required that we test $H_0: \mu = 400$ against $H_1: \mu = 520$

i. Show that a test procedure is to reject H_0 $if\ X > k$ (4 Marks)

ii. Obtain k such that the probability of coming type 1 error is 0.05. (3 Marks)

iii. What is the power of the test. (3 Marks)

c) The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested and the following results were obtained;

108, 124, 124, 106, 115, 138, 163, 159, 134, 139

i. We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypothesis for investigating this claim. (1 Marks)

ii. Test these hypothesis using $\propto = 0.01$. What are your conclusions? (4 Marks)

iii. Construct a 99% confidence interval on the mean shelf life. (2 Marks)

QUESTION FOUR (20 MARKS)

a) Given the general linear model

$$Y_i = \propto + \beta + \varepsilon_i \qquad i = 1,2, \cdots, n$$

Where ε_i are the error terms such that

$E(\varepsilon_{i)} = 0, Cov(\varepsilon_i \varepsilon_j) = 0 \quad i \neq j$ and variance $(\varepsilon_i) = \delta^2\ \varepsilon_i \sim N(0, \delta^2)$

i. Obtain the normal equation using the method of maximum likelihood. (4 Marks)

ii. Find the estimates for $\propto, \beta\ and\ \delta^2$ (3 Marks)

iii. Use the model $Y_i = \propto + \beta + \varepsilon_i$ to test the hypothesis (3 Marks)

$$H_0: \beta_1 = 0\ Vs$$

$$H_1: \beta_1 \neq 0$$

b) Two Chemicals treatments A and B were applied to a random sample of seeds. After treatment, germination tests were conducted and the following results were observed.

Treatment	No. of Seeds that germinated	No. of seeds that did not germinate	Total
A	120	30	150
B	110	15	125
Total	230	45	275

Does the data indicate that the chemical treatment differ in the effect on germination at 5% significant level. (10 Marks)