MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

SECOND YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

AMS208: PROBABILITY AND DISTRIBUTION THEORY II

DURATION: 2 HOURS

DATE: 18TH APRIL 2018

TIME: 2.00PM – 4.00PM

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

a) Differentiate between a parameter and a statistic as used in sampling theory. (2 Marks)

b) Let $x_1, x_2, \cdots, x_n$ be random variables each with mean μ and standard deviation $\sigma > 0$. Assume that each pair of x_i, x_j of random variables with $i \neq j$ is uncorrelated. Let

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{X})^2$$

be the sample variance. Show that $E(S^2) \neq \sigma^2$ (5 Marks)

c) A random sample of size 10 is taken from a normally distributed population with unknown variance. The sample mean $\bar{X}$ and standard deviations are given by 38.4 and 5.5. Calculate the 98% confidence interval for the variance (σ^2). (4 Marks)

d) Let $x_1, x_2, \cdots, x_n$ be a random sample size n taken from a probability distribution where S^2 is the population variance $(\sigma^2 > 0)$. Show that $E(S^2) = \frac{n-1}{n}\sigma^2$. (4 Marks)

e) State and prove the weak law of large numbers. (5 Marks)

f) Let $x_1, x_2, \cdots, x_n$ be a sample from a normal population with mean μ and variance σ^2. Show that the variable $\frac{n-1}{\sigma^2}S^2$ has a chi-square distribution with $n - 1$ degrees of freedom. (5 Marks)

g) Let X be a continuous random variable with the following p.d.f.

$$f(x) = \begin{cases} 5x^4 & for\ 0 < x < 1 \\ 0 & Otherwise \end{cases}$$

i. Find the probability density function of $Y = X^3$ (3 Marks)

ii. Determine $P(\frac{1}{3} < y < 2)$ (2 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Let x_1, x_2, x_3, x_4 be four independent sample observations of Poisson distribution with parameter θ. Show that

$$T = \frac{3x_1 + 2x_2 + 3x_3 + 2x_4}{10}$$

is unbiased estimator of θ. (3 Marks)

b) The joint density of X_1 and X_2 is given by

$$f(x_1, x_2) = \begin{cases} 6e^{-3x_1 - 2x_2}, & 0 < x_1, < x_2, 0 < x_2 < \infty \\ 0 & Otherwise \end{cases}$$

Find the density function of Y where $Y = X_1 + X_2$ by distribution function technique.

(6 Marks)

c) A population consists of the five numbers 2,3,6,8 and 11. Consider all the possible samples of size two which can be drawn with replacement from this population. Find;

i. The mean of the population. (1 Mark)

ii. The standard deviation of the population. (3 Marks)

iii. The mean of the sampling distribution of the mean. (4 Marks)

iv. The standard deviation of the sampling distribution of the means. (3 Marks)

QUESTION THREE (20 MARKS)

a) State and prove the central limit theorem for independent and identically distributed random variables. (9 Marks)

b) The time taken to complete a certain test is assumed to be normally distributed with a mean μ and variance σ^2. A random sample of size 15 is taken and the following observed. 119.2, 128.9, 112.8, 120.9, 91.1, 115.3, 111.9, 92.0, 181.5, 123.6, 146.2, 99.0, 131.7, 141.7 and 114.9. Find the 90% confidence interval for the variance. (5 Marks)

c) A random sample of 16 value form a normal population showed a mean of 41.5cm and the sum of squares of the deviations from this mean equal to 125cm^2. Obtain the 95% and 99% confidence limits for the population mean. (6 Marks)

QUESTION FOUR (20 MARKS)

a) Let $x_1, x_2, \cdots, x_n$ be random variables distributed normally and independently with mean μ_i and variance (σ^2) where $i = 1, \cdots, n$. Show that

$$U = \sum_{i=1}^{n} \left(\frac{x_i - \mu_i}{\sigma}\right)^2$$

has a Chi-square distribution with n degrees of freedom. (6 Marks)

b) Let U_1 and U_2 be independent random variables where U_1 has a Chi-square distribution with $\sqrt{2}$ degrees of freedom. Derive the probability distribution of the random variable

$$F = \frac{U_1}{\sqrt{1}} \Big/ \frac{U_2}{\sqrt{2}}$$

(14 Marks)