MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

FIRST YEAR **SECOND** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

AMS107: PROBABILITY AND DISTRIBUTION THEORY I

DURATION: 2 HOURS

DATE: 26TH APRIL 2018

TIME: 8.00AM – 11.00AM

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

a) Define the following terms as used in probability and statistics

i. Random variable (1 Mark)

ii. Probability distribution (1 Mark)

iii. Random Vector (1 Mark)

iv. Correlation coefficient (1 Mark)

b) If x is a discrete random variable

i. From a Bernoulli distribution given as

$$f(x,\theta) = \begin{cases} u^x(1-u)^{1-x} & x = 0,1 \\ 0 & otherwise \end{cases}$$

Derive the moment generating function of x (4 Marks)

ii. Given a random variable x a POPn

$$f(x) = \begin{cases} 1\ for - {}^1\!/_2 < x < \theta + {}^1\!/_2 \\ 0 \end{cases}$$

Obtain the expected value of x. (4 Marks)

c) i) Define the moment generating function of a continuous random variable$x$. (2 Marks)

Hence or otherwise obtain the moment generating function of a random variable $X \sim N(0,1)$ (5 Marks)

ii) Obtain the moment generating function of a random variable Y which is normal with the mean μ and standard deviation δ using results of (i) above. (4 Marks)

d) suppose X and Y are two continuous random variable with a bivariate probability density function given by

$$f_{xy}^{(x,y)} = \begin{cases} x = y & if\ \ 0 \le x, y < 1 \\ 0 & elsewhere \end{cases}$$

i. Show that

$$\iint_{IR} f_{xy}^{(x,y)} dxdy = I$$

(3 Marks)

ii. Obtain the marginal probability density of $X(f_x^{(x)}$ (4 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Define the stochastic independence of two random variables (3 Marks)

b) Let X and Y have C bivariate probability distribution given as

$$f_{xy}^{(x,y)} = \begin{cases} 4xy + 2x + 2y + {}^1\!/_4 & if\ 0 \le x, y < 1 \\ 0 & Otherwise \end{cases}$$

i. Obtain the marginal distribution of X and Y (4 Marks)

ii. Determine whether X and Y are stochastically independent. (4 Marks)

c) Suppose X and Y are two independent standard normal random variables, determine the joint probability density of X and Y i.e. $f_{xy}^{(x,y)}$ (7 Marks)

QUESTION THREE (20 MARKS)

a) State any two properties of a gamma distribution. (4 Marks)

b) A random variable X has a gamma distribution of its probability distribution given as

$$f(t) = \begin{cases} \frac{\lambda^{\propto}}{(\propto)} t^{\propto - 1} e^{-\lambda t} & for \begin{matrix} t \geq 0 \\ \propto, \lambda > 0 \end{matrix} \\ 0 & elsewhere \end{cases}$$

i. Obtain the expectation and variance of X by direct method (or using pdf)(12 Marks)

ii. Obtain the moment generating function of X (4 Marks)

QUESTION FOUR (20 MARKS)

a) Let ???? be a random variance with a density function

$$f(x) = \begin{cases} x^3/_3 & for -1 < x < 2 \\ 0 & Otherwise \end{cases}$$

Find the expected value of $g(x) = 4x + 3$ (5 Marks)

b) A continuous random variable X has a moment generating function given by

$$M_X(t) = \frac{9}{(3-t)^2}$$

Find the mean and variance of X (6 Marks)

c) A random variable X is said to have a chi-square distribution with n-degrees of freedom if its mgf (moment generating function) is defined for any $t < 1/_2$ and it's equal to $M_x(t) = (1-2t)^{-n/_2}$. If $Y = X_1 + X_2$ where X_1 and X_2 are independent random variables have chi-square distribution with n_1 and n_2 degrees of freedom respectively. Prove that X has chi-square with $n_1 + n_2\ dof$. (9 Marks)