



# MURANG'A UNIVERSITY OF TECHNOLOGY

## SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**RE-SIT EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS  
AND COMPUTER SCIENCE**

AMM2206: CALCULUS III

DURATION: 2 HOURS

DATE: 25<sup>TH</sup> APRIL 2018

TIME: 2.00PM – 4.00PM.

### **Instructions to Candidates:**

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

**SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)**

**QUESTION ONE**

- a) i) Define a series (1 Mark)  
ii) Give two types of series. (2 Marks)
- b) Determine whether the series below converges or diverges.
- i.  $\sum_{k=1}^{\infty} \frac{3}{4^k}$  (2 Marks)
- ii.  $\sum_{k=1}^{\infty} \frac{1}{k^2}$  (2 Marks)
- c) Find the Taylor polynomials  $P_1(x)$ ,  $P_2(x)$  and  $P_3(x)$  for  $\sin x$  about  $x = \pi/2$  (6 Marks)
- d) Prove the following trigonometric identities of complex functions
- i.  $\sin(-z) = -\sin z$  (2 Marks)
- ii.  $\tan z = \frac{\sin z}{\cos z}$  (2 Marks)
- e) Work out the following double integral;
- $$\int_0^1 \int_1^2 (1-x)y^2 dx dy$$
- (4 Marks)
- f) Obtain a reduction formulae for  $I_n = \int \cos^n x dx$ . (6 Marks)  
Hence evaluate  $\int \cos^3 x dx$ . (3 Marks)

**SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION**

**QUESTION TWO (20 MARKS)**

- a) i) What are improper integrals? (1 Mark)  
ii) Evaluate the following integral and find out whether it converges or diverges
- $$\int_{-\infty}^0 e^x dx$$
- (4 Marks)
- b) i) Use Lagrange multipliers to show that  $f(x, y, z)$  has only one critical point on the surface  $x^2 + y^2 - z = 0$  (6 Marks)  
ii) Show that the one critical point is a minimum. (4 Marks)
- c) Give the Taylor series expansion for  $\frac{1}{x}$  upto fifth degree about  $x = 1$ . (5 Marks)

**QUESTION THREE (20 MARKS)**

- a) Find the  $n^{\text{th}}$  Maclaurin Polynomial for  $\cos x$  at  $x = 0$ . (10 Marks)
- b) Obtain a reduction formulae for
- $$I_n = \int x^n e^x dx$$
- (4 Marks)
- Hence evaluate
- $$\int_0^2 x^3 e^x dx$$
- (6 Marks)

**QUESTION FOUR (20 MARKS)**

a) Use integration by parts to compute the following integrals;

i.  $\int x e^x dx$  (2 Marks)

ii.  $\int x \sqrt{x^2 + 3} dx$  (2 Marks)

b) Prove the following hyperbolic representation of complex function

$$\sinh(z_1 + z_2) = \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2$$

(6 Marks)

c) Evaluate the following improper integral and find out whether it converges or diverges

$$\int_{-1}^1 \frac{dx}{x^3}$$

(4 Marks)

d) Evaluate the following double integral

$$\int_0^2 \int_3^4 (x - y)x^2 dx dy$$

(6 Marks)

**QUESTION FIVE (20 MARKS)**

a) i) Define a sequence. (1 Mark)

ii) Give two types of sequences (2 Marks)

iii) Work out the following sequences to determine whether it converges or diverges

$$\lim_{n \rightarrow \infty} \left[ \frac{n}{2n + 1} \right]_{n=1}^{\infty}$$

(4 Marks)

b) Prove the following trigonometric identity of complex functions

$$e^{iz} = \cos z + i \sin z$$

(4 Marks)

c) Apart from trigonometric and hyperbolic functions, give three other elementary complex functions. (3 Marks)

d) Compute

$$\int_0^2 \int_0^{y/2} \int_0^{\sqrt{y^2 - x^2}} z dz dy dx$$

(6 Marks)