



MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**FIRST YEAR SECOND SEMESTER EXAMINATION FOR BACHELOR OF
SCIENCE IN ACTUARIAL SCIENCE**

AMC102 – FINANCIAL MATHEMATICS I

DURATION: 2 HOURS

DATE: 25TH APRIL 2018

TIME: 2.00PM – 4.00PM

Instructions to Candidates:

1. Answer **Section A** and **Any Other Two** questions in **Section B**.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION (30 Marks)

QUESTION ONE

- a) Differentiate between interest rate and discount rate as used in finance. (2 Marks)
- b) The rate of interest of a certain bank deposit account is 4.5% per annum effective. Find the accumulated value of \$5000 deposited today in this account after 7 years. (2 Marks)
- c) Given that $d^{(4)} = 0.06$. Find the values of $i, v, i^{(6)}$ and δ leaving your answers in 4 s.f. (6 Marks)
- d) The force of interest payable on a savings account is

$$\delta(t) = \begin{cases} 0.04 & , & 0 \leq t \leq 5 \\ 0.01(t^2 - t) & , & t > 5 \end{cases}$$

Calculate the present value of 1500 payable at time $t = 10$. (5 Marks)

- e) A project requiring an initial outlay of \$1200 is expected to generate returns of \$400 in years 1 and 2, and \$350 in years 3 and \$4. If the Net Present Value (NPV) is \$22 at 9% interest rate and -\$4 at 10% interest rate, find the Internal Rate of Return (IRR) for the project. (4 Marks)
- f) An investment requires an initial outlay of \$10,000 and follow-up annual payments of \$1000 at the end of each of the first 10 years. Starting from the end of the 11th year, the investment returns 5 equal annual payments of amount \$X. Determine the amount X that could yields an effective rate of 10% p.a. over the next 15 year period. (5 Marks)
- g) Consider an annuity of payments of \$1000 at the end of every second year. What is the present value of this annuity if it runs for 10 years and the interest is 7% p.a.? (3 Marks)
- h) Compute the nominal interest rate per annum convertible monthly that is equivalent to the simple interest of 7% p.a. effective over a period of three months. (3 Marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

- a) How long does it take for \$900 to accumulate to \$1000 under an interest rate of 4% p.a. given that interest is convertible
- Monthly (2 Marks)
 - Weekly (2 Marks)
 - Explain why your answers in (i) and (ii) above differ. (1 Mark)
- b) What do you understand by the phrase “Time value of money” as used in finance? How do banks and other money-lending institutions account for the time value of money?(4 Marks)
- c) A 20 year loan of \$50,000 is repaid as follows;
The borrower pays only interest on the loan, annually in arrears at the rate of 5.5% p.a.

The borrower will take out a separate savings policy which involves making monthly payments in advance such that the proceeds will be sufficient to repay the loan at the end of its term

The payments into the savings policy will accumulate at a rate of interest of 4% p.a. effective.

Required:

Compute the monthly payments into the savings account which ensures that the account contains \$50,000 after 20 years. Write down the equation of value for the effective rate of interest on the loan if it is repaid using this arrangement. (7 Marks)

- d) An annuity is payable in arrears for 10 years. The first payment is of amount \$8,000 and the value of each subsequent payment decreases by \$300 each year. Find the present value of the decreasing annuity on the basis of an interest rate of 5% per annum. (4 Marks)

QUESTION THREE (20 MARKS)

- a) The force of interest payable on a savings account is

$$\delta(t) = \begin{cases} 0.06 & , 0 < t \leq 6 \\ 0.05 + 0.0002t^2 & , t > 6 \end{cases}$$

Calculate the accumulated value at time $t = 12$ of a continuous payment stream of \$100 per annum payable from time $t = 0$ to time $t = 8$. (8 Marks)

- b) In return for a loan of &100, a borrower agrees to repay \$110 after seven months.
- Find the rate of compound interest per annum (2 Marks)
 - Find the compound discount rate equivalent to the interest rate in part (i). (2 Marks)
 - Shortly after receiving the loan, the borrower requests that he be allowed to repay the loan by repayment of \$50 on the original date of settlement, and a second payment 6 months after this date. Assuming that the lender agrees to honour this request, and that the calculation is made on the original interest basis; find the amount of the second payment under a revised transaction. (4 Marks)
- c) On 15th November in each of the years 1964 to 1979 inclusive, an investor deposited \$500 in a special bank savings account. ON 15th November 1983, the investor withdrew all his savings. Given that the entire period the bank used an annual interest rate of 7% convertible quarterly for its special savings account. How much did the investor withdraw?(4 Marks)

QUESTION FOUR (20 MARKS)

- a) Distinguish between the following types of annuities as used in finance;
- i. Annuity immediate (1 Mark)
 - ii. Annuity due (1 Mark)
 - iii. Annuity certain (1 Mark)
 - iv. Perpetuity (1 Mark)
- b) \$50 is paid at the end of each year forever into an account that earns an effective rate of interest of 10% p.a. Given that the first payment is made six years from now, find the present value of the investment. (4 Marks)
- c) A four-year loan of \$5,000 is repaid by equal annual payments at the end of each year. Compute the annual payment on the basis of an interest rate of 6% p.a. and draw up a loan schedule, showing the interest component and capital component of every payment as well as the outstanding balance. (7 Marks)
- d) An investor is contemplating two investment projects. Project A requires an initial payment of \$10,000 in return for which the investor will receive \$250 at the end of every quarter for 15 years.
- Project B requires an initial payment of \$11,000. In return the investor will be paid \$605 at the end of every year for 18 years and the initial payment of \$11,000 will be repaid at the end of 18 years. Which is the preferred investment? (5 Marks)

QUESTION FIVE (20 MARKS)

- a) Define the following types of cash flow models;
- i. Zero-coupon bond (1 Mark)
 - ii. Mortgage loan (1 Mark)
 - iii. Sinking fund (1 Mark)
- b) A fund has a value of \$150,000 on 1st July 2006. A net cash flow of \$30,000 was received on 1st July 2007 and a further net cash flow of \$40,000 was received on 1st July 2008. The fund had a value of \$150,000 on 30th June 2007 and a value of \$225,000 on 30th June 2008. Given that the fund value was \$280,000 on 1st January 2009, Calculate;
- i. The time-weighted rate of return (TWRR) per annum earned on the fund between 1st July 2006 and 1st January 2009. (4 Marks)

- ii. The money weighted rate of return (MWRR) per annum earned on the fund between 1st July 2006 and 1st January 2009. (5 Marks)
 - iii. Explain why the TWRR is more appropriate than MWRR when comparing the performance of two investment managers over the same period of time. (2 Marks)
- c) A bank offers two repayment alternatives for a loan that is to be repaid over 10 years. The first requires the borrower to pay \$1200 per annum quarterly in advance, while the second requires the borrower to make payments at an annual rate of \$1260 every second year in arrears. Determine which terms would provide the best deal for the borrower at a rate of 4% p.a. effective. (5 Marks)