MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

THIRD YEAR **SECOND** SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE

AMM 316 – ORDINARY DIFFERENTIAL EQUATIONS II

DURATION: 2 HOURS

DATE: 17TH APRIL, 2018

TIME: 2.00 – 4.00 P.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

a) Find the Wronskian of the functions $f(x) = e^{ax}$ Cos bx and $g(x) = e^{ax}$ Sin bx, given that 'a' and 'b' are constants and $b \neq 0$ (5 marks)

b) Show that the functions $f_1 = 9 \text{ Cos } 2x$ and $f_2 = 2 \text{ Co}s^2 x - 2 \text{ Sin}^2 x$ are linearly dependent (4 marks)

c) Write the following 4^{th} order differential equation as a system of first order linear differential equations and present the system in matrix form

$\frac{d^4y}{dx^4} - 7\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$ (5 marks)

d) Determine the eigen values and corresponding eigen vectors of the matrix $A = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix}$ (6 marks)

e) Reduce Bessel's equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ to its normal form (5 marks)

f) Identify and classify the singular points of the differential equation $x(1-x)y'' + xy' + y = 0$ (5 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

a) Given that $f = e^{-2x}$ Cos 3x and $g = e^{-2x}$ Sin 3x,

   i. Show that $W[f,g] \neq 0$ (4 marks)

   ii. Verify that f and g are solutions of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$ and write the corresponding complementary function (4 marks)

b) i. Show that the differential equation $x^3\frac{d^3y}{dx^3} - 6x\frac{dy}{dx} + 12y = 0$ has three linearly independent solutions of the form $y = x^r$ (8 marks)

   ii. Verify that $y = \ln x$ is a particular integral of the differential equation $x^3\frac{d^3y}{dx^3} - 6x\frac{dy}{dx} + 12y = 12\ln x - 4$ obtained from Q2b(i) above and hence write its general solution (4 marks)

QUESTION THREE

a) Convert the differential equation y'' + y' – 6y = 0; y(0) = 0; y'(0) = 5 into a first order system of linear differential equations and write the system in matrix form (5 marks)

b) Find the solution of the following initial value problem $\underline{x}^1 = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \underline{x}, \underline{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (15 marks)

QUESTION FOUR

a) Find the indicial equation and recurrence relation for Bessel's differential equation

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ (12 marks)

b) Solve the Bernoulli equation $y' = \frac{2}{x}y + y^2$ (8 marks)