# MURANG'A UNIVERSITY OF TECHNOLOGY

## SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**THIRD** YEAR **SECOND** SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH PROGRAMMING)

AMM 312 – METHODS

DURATION: 2 HOURS

DATE: 23RD APRIL, 2018

TIME: 9.00 – 11.00 A.M.

**Instructions to Candidates:**

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

## SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

**QUESTION ONE**

a) Show that the function

$f(x) = \begin{cases} x^{-3/2} & 0 < x < 1 \\ 0 & elsewhere \end{cases}$ is absolutely integrable (2 marks)

b) Evaluate $\int_0^\infty \frac{Sin\ t}{t}\ dt$ (2 marks)

c) Find the half-range Fourier sine series representation of the function

$f(t) = \begin{cases} 3t & 0 < t < 2 \\ f(t+4) & \end{cases}$ (6 marks)

d) Use Laplace transforms of the second derivative to find $\mathcal{L}\{Sin\ 2t\}$ (5 marks)

e) Find the Fourier Cosine transform of the function $f(t) = e^{-t}$ (6 marks)

f) Use the convolution theorem to find $\mathcal{L}^{-1}\left\{\frac{1}{s^3 + 4s}\right\}$ (5 marks)

g) A system at rest has a constant input $f(t) = 5$ applied when t = 0. The output of the system is found to be $x(t) = 1 - cost$

Find the impulse response function of this system (4 marks)

## SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

**QUESTION TWO**

a) Use first principles to find the Laplace transform of the function

$f(t) = (t+1)e^{2t}$ (5 marks)

b) Find the transfer function and the impulse response function of a system which is characterized by the differential equation $\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 10x = Cos\ 2t,$

$x(0) = x'(0) = 0$ (5 marks)

c) Use Laplace transforms to find the solution to the initial value problem

$2\frac{d^2x}{dt^2} - \frac{dx}{dt} - 3x = 2 + e^{2t}$ given the $x(0) = 0$ and $x^1(0) = 1$ (10 marks)

**QUESTION THREE**

a) Find the half-range Fourier cosine series of the function $f(x) = \begin{cases} 1 - x & 0 < x < 1 \\ f(x+2) & \end{cases}$

(8 marks)

b) A periodic function is given by $f(t) = \begin{cases} t & 0 \le t < \pi \\ 0 & \pi \le t < 2\pi \\ f(t+2\pi) & \end{cases}$ (8 marks)

i. Obtain its Fourier series (9 marks)

ii. By substituting t = π into the series obtained in part (i) above, show that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

(3 marks)

**QUESTION FOUR**

a) A function f(t) is defined by $f(t) = \begin{cases} 1 & -3 < t < 3 \\ 0 & elsewhere \end{cases}$

Show that its Fourier transform is given by

$\bar{F}(w) = \sqrt{\frac{2}{\pi}} \left(\frac{Sin 3w}{w}\right)$ (5 marks)

b) Find the Fourier sine transform of the function

$f(t) = e^{-2t}$

Hence show that $\int_0^{\infty} \frac{wSinwt}{w^2+4} dw = \frac{\pi}{2} e^{-2t}$ (7 marks)

c) Use the method of differentiation with respect to a parameter to show that

$\int_0^1 \frac{x^2-1}{lnx} dx = \ln 3$ (8 marks)