



MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**THIRD YEAR SECOND SEMESTER EXAMINATION FOR BACHELOR OF
SCIENCE MATHEMATICS AND ECONOMICS**

AMM 308 – COMPLEX ANALYSIS I

DURATION: 2 HOURS

DATE: 20TH APRIL, 2018

TIME: 2.00 – 4.00 P.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

- a) State the residue theorem (1 mark)
- b) Given the complex numbers
 $Z_1 = 2 + i$, $Z_2 = 3 - 5i$ and $Z_3 = \frac{1}{Z_1} + \frac{2}{Z_2}$
Express Z_3 in the form $a + bi$ (2 marks)
- c) Find the principal value of $\ln(7+2i)$ (4 marks)
- d) Evaluate $\int_{8+\pi i}^{8-3\pi i} e^{(z/2)} dz$ (3 marks)
- e) Find $(1+2i)^{10}$ (3 marks)
- f) Find the image of the line $Z = x + 2i$ under the transformation $w = Z^2 + 3$ (4 marks)
- g) A complex valued function w is defined by $w = (3x - 2xy) + i(x^2 - y^2 + 3y)$. determine $\frac{dw}{dz}$ (4 marks)
- h) Evaluate $\oint_c \frac{e^z}{z^2 - 4} dz$ where c is the circle $|z - 1| = 2$ (4 marks)
- i) Show that the function $f(z) = e^{\bar{z}}$ is not analytic where \bar{z} is the complex conjugate of z (5 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

- a) Classify the singularity of the function $f(z) = \frac{1}{\sin z - z}$ at $z = 0$ (6 marks)
- b) Obtain the Laurent Series for the function
 $f(z) = \frac{4z-6}{z(z-1)(z-3)}$ in the annulus $1 < |z - 1| < 3$ (8 marks)
- c) Find the Möbius transformation that maps $z = 1, 0, i$ onto $w = -1, \infty, 1$ respectively (6 marks)

QUESTION THREE

- a) Show that the functions $u = \left(r + \frac{1}{r}\right) \cos\theta$ and $v = \left(r - \frac{1}{r}\right) \sin\theta$ satisfies the Cauchy-Riemann equations (6 marks)
- b) Show that the function $u(x,y) = 2x^2 - 2y^2 - 4x$ is harmonic hence find its conjugate harmonic function $v(x,y)$ such that $f(z) = u+iv$ is analytic (6 marks)
- c) Find the image of the circle $|z|=2$ under the transformation $w = \frac{z+2i}{z+i}$ (8 marks)

QUESTION FOUR

a) Use the residue theorem to evaluate the integral $\oint_c \frac{\cos z}{(z+\pi/2)^2} dz$ where c is the circle $|z+i|=2$

(6 marks)

b) Evaluate $\int_{-\infty}^{\infty} \frac{e^{\pi z}}{z^2+9} dz$

(6 marks)

c) Evaluate $\int_0^{2\pi} \frac{1+4\cos\theta}{17-8\cos\theta} d\theta$

(8 marks)