



MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**SECOND YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF
BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE**

AMM 304 – ORDINARY DIFFERENTIAL EQUATIONS I

DURATION: 2 HOURS

DATE: 24TH APRIL, 2018

TIME: 2.00 – 4.00 P.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

- a) Define the following terms to a differential equation;
- Order
 - Degree
 - Particular Solution (3 marks)
- b) Determine the differential equation whose general solution is given by $y = A + Be^{4t}$ (3 marks)
- c) i. Define a homogenous function $f(x,y)$ (1 mark)
- ii. Determine if the function $f(x,y) = \frac{x^2 - 2xy + 3y^2}{x^3 + xy^2}$ is homogenous (2 marks)
- iii. Solve the differential equation $y^2 e^x + \frac{dy}{dx} = y^2$ given the initial conditions that $y=1$ when $x=0$ (4 marks)
- d) The distance x of a particle from a fixed point p in time t is given by the equation
- $$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 0$$
- Determine the distance x from p at any time t (3 marks)
- e) For certain values of the constants p, q and r the function, $y = p \sin 2x + q \cos 2x + r$ is a solution of the equation $y'' + 2y' + y = 25 \cos 2x + 3$. Find the values of the constants (4 marks)
- f) The rate of increase of the population of a certain town is proportional to the population. If the population of the town was 40,000 in 1980 and was 100,000 in 2016 determine the expected population size in 2030 (5 marks)
- g) i. Show that $(4x + 3y^2)dx + 2xy dy = 0$ is not exact (1 mark)
- ii. Find an integrating factor of the form x^n where n is a positive integer (2 marks)
- iii. Using your integrating factor found in (ii) to solve the equation
- $$(4x + 3y^2)dx + 2xy dy = 0$$
- (2 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

- a) i. Prove that the first order differential equation $M(x,y)dx + N(x,y)dy = 0$ is exact if
- $$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$
- (4 marks)
- ii. Determine if the following differential equation is exact or not
- $$(x^2 + 2xy + y)dx + (y^3 + x^2 + x)dy = 0$$
- Hence solve the equation (5 marks)

- b) The differential equation $y'' + 2y' - 3y = Q(x)$ is to be solved using the method of undetermined coefficients to find the particular solution y_p
- Find the homogenous solution (2 marks)
Without solving for y_p write down the general form of y_p if $Q(x)$ is
 - $\sin x$ (1 mark)
 - xe^{-3x} (1 mark)
- c) Determine an integrating factor to the following equation and hence solve it
 $\{(x + 1)^4 + 2y\}dx - (x + 1)dy = 0$ (7 marks)

QUESTION THREE

- a) Let $F(D)$ be the function of the differential operator associated with the differential equation
 $ay'' + by' + cy = Q(x)$
- Show that $F(D) = F(K)e^{kx}$ (2 marks)
 - Use (i) to solve $3y'' - 11y' + 6y = e^{4k}$ (4 marks)
- b) Reduce the following differential equation to first order and hence solve
 $(1 - x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = x$ (7 marks)
- c) Use the method of undetermined coefficients to solve the differential equation
 $\frac{d^2y}{dx^2} + y = 5 \cos x + 2 \sin x$ (7 marks)

QUESTION FOUR

- a) Use a suitable change of variable to transform the following equation to a linear form and hence solve it
 $\frac{dy}{dx} + \frac{1}{x}y = x^2y^3$ (6 marks)
- b) i. Define the inverse of the differential operator D (2 marks)
 ii. Determine $(1 + D)^{-1}(x^2 + 2x + 1)$ (3 marks)
- c) i. Obtain the power series solution of the same equation, $\frac{dy}{dx} = y$ in ascending powers of x (6 marks)
 ii. Solve the equation $\frac{dy}{dx} = y$ using the method of separation of variables (3 marks)
 iii. Verify that the two solutions are equivalent (1 mark)