



MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

**SECOND YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF
BACHELOR OF SCIENCE IN MATHEMATICS & ECONOMICS
BACHELOR OF SCIENCE MATHEMATICS & COMPUTER SCIENCE AND
BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING**

AMM 203 – VECTOR ANALYSIS

DURATION: 2 HOURS

DATE: 23RD APRIL, 2018

TIME: 2.00 – 4.00 P.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

a) Given the vector $\bar{A} = 3\underline{i} + 4\underline{j} - 5\underline{k}$

Find the direction cosines hence verify the vector direction cosine law (4 marks)

b) Find the orthogonal projection of the vector $\bar{A} = 2\underline{i} + \underline{j} - 5\underline{k}$ on $\bar{B} = 3\underline{i} - \underline{j} + 4\underline{k}$ (4 marks)

c) Given the vector $\bar{A} = \underline{i} + \underline{j} + \underline{k}$ and $\bar{C} = \underline{j} - \underline{k}$ Find another vector \bar{B} such that

$$\bar{A} \times \bar{B} = \bar{C} \text{ and } \bar{A} \cdot \bar{B} = 3 \quad (6 \text{ marks})$$

d) A particle under the action of a constant force $\bar{F} = \underline{i} + 3\underline{j} + 2\underline{k}$ is displaced from a point

A(-1, 2, 4) to point B(4, 6, 3). Determine the work done by this force (3 marks)

e) Use divergence theorem to show that $\int_S (ax\underline{i} + by\underline{j} + cz\underline{k}) \cdot \bar{n} ds = \frac{32}{3}\pi(a + b + c)$ where a,

b & c are constants and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ (5 marks)

f) Evaluate $\int_C \phi d\bar{r}$ between A(0, 0,0) and B(2,1,5) where $\phi = xyz$ and $x = 2t, y = t^2$ & $z = 5t$

(4 marks)

g) Find a unit normal vector to the surface $\phi(x, y, z) = x^2y + y^2z^2$ at a point (1, -1, 2) (4 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

a) Given the scalar function $\phi = \phi(x, y, z)$ prove that $\text{curl}(\text{grad } \phi) = 0$ (3 marks)

b) Find the directional derivative of $\phi(x, y, z) = x^2y^3 - 3yz^3$ at a point (1, -1, 1) in the direction of the vector $\bar{A} = 2\underline{i} - 3\underline{j} - \underline{k}$ (5 marks)

c) A particle moves in space along a curve whose parametric equations are $x = 2t^3 + t, y = t^2 - 5$ and $z = 4t^2 + 2t$. find the components of its velocity and acceleration at an instant when $t = 2$ sec in the direction of the vector $\bar{A} = 3\underline{i} + 2\underline{j} - \underline{k}$ (6 marks)

d) Given the space curve c whose position vector is $\bar{r} = 2\cos t \underline{i} + 2\sin t \underline{j} + 4t \underline{k}$ Find the unit tangent and the unit normal vectors (6 marks)

QUESTION THREE

a) Given the vector point function $\bar{F} = 2x^2y^3 \underline{i} + (x^2 + z^2)\underline{j} + y^2z^3 \underline{k}$

Find:

- i. $\text{Div } \bar{F}$ and
 - ii. $\text{Curl } \bar{F}$ at $(1, 1, 2)$ (5 marks)
- b) Find the circulation of the velocity vector $\bar{V} = \underline{i}(2x + y^2) + \underline{j}(3y - 4x)$ along the parabola $y = x^2 + 1$ from $(0, 1)$ to $(2, 5)$ (7 marks)
- c) Show that the force field defined by $\bar{F} = (3x^2z + e^y)\underline{i} + (2yz^2 + xe^y)\underline{j} + (x^3 + 2y^2z)\underline{k}$ is a conservative force field hence evaluate $(1, 3, 5)$
 $(0, 1, 1)$
 $\int (3x^2z + e^y)dx + (2yz^2 + xe^y)dy + (x^3 + 2y^2z)dz$ (8 marks)

QUESTION FOUR

- a) Find the work done by the force $\bar{F} = 2xy\underline{i} + y\underline{j} - z\underline{k}$ in moving a particle around the circle $x^2 + y^2 = 9$ once (6 marks)
- b) Use Green's theorem to evaluate $\oint_c [(3x^2 - y^2)dx + (4y - 6xy)dy]$ where c is the boundary of the region bounded by the curves $y = x^{1/2}$ and $y = x^2$ (7 marks)
- c) Apply the Gauss-Divergence theorem to evaluate $\int_s \bar{F} \cdot d\bar{s}$ where $\bar{F} = 4xz\underline{i} - y^2\underline{j} + yz\underline{k}$ and s is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ (7 marks)