MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

SECOND YEAR **SECOND** SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN SOFTWARE ENGINEERING
AND
FIRST YEAR **SECOND** SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE,

AMM 201 – LINEAR ALGEBRA I

DURATION: 2 HOURS

DATE: 23RD APRIL, 2018

TIME: 9.00 – 11.00 A.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

a) Determine the values of $\propto$ such that $A = \propto i - 2j + k$ and $B = 2 \propto i + \propto j + 4k$ are perpendicular (3 marks)

b) Find the distance from (2,1-,3) to a plane 2x−2y−z = 9 (2 marks)

c) Determine whether S-{(1,0,1), (2,1,2), (1,2,2)} is a basis of $\mathbb{R}^3$ (4 marks)

d) Determine whether th transformation T: $\mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by T(x,y) = (x-2y, y, y+x) is a linear transformation (3 marks)

e) Let V=$\mathbb{R}^3$. Show that w={(a,o,c)|a∈ $\mathbb{R}$} is a subspace of V (3 marks)

f) Determine whether the following vectors are linearly independent or dependent $\underline{a}$ = (1,1,-1) $\underline{b}$ = (2,-3,1) $\underline{c}$ = (8,-7,1) (4 marks)

g) Use row operations to find the inverse of the matrix $B = \begin{pmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{pmatrix}$ (4 marks)

h) Determine the null space and hence the nullify of the matrix $C = \begin{pmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ -6 & 3 & -9 \end{pmatrix}$ (4 marks)

i) Define orthogonal vectors and determine whether the vectors U = (3,4,-1) and V = (1,0,3) are orthogonal with respect to Euclidean inner product space (4 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

a) Use matrices to find a general solution to system

$x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 = 2$

$3x_1 - 9x_2 + 7x_3 - x_4 + 3x_5 = 7$

$2x_1 - 6x_2 + 7x_3 + 4x_4 - 5x_5 = 7$ (5 marks)

b) The vectors U=(1,2,1), V=(3,2,0) and W=(1,2,5) form a basis S for Euchldean space $\mathbb{R}^3$. find the matrix A that represents the inner product in $\mathbb{R}^3$ relative to the basis S (6 marks)

c) Determine the range and rank of the matrix $A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 4 \\ 0 & 1 & 1 & 0 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ (4 marks)

d) Write V=(3,7,4) as a linear combination of $U_1 = (1,2,3)$ $U_2 = (2,3,7)$ and $U_3 = (3,5,6)$

(5 marks)

QUESTION THREE

a) Let F: $\mathbb{R}^5 \rightarrow \mathbb{R}^3$ be a linear transformation defined by

$F(x_1, x_2, x_3, x_4, x_5) = (x_1 + 2x_2 - 3x_3 - 2x_4 + 4x_5,\ \ 2x_1 + 5x_2 - 8x_3 - x_4 + 6x_5,$

$x_1 + 4x_2 - 7x_3 + 5x_4 + 2x_5)$. Find a basis and dimension of the kernol of map F (10 marks)

b) Use the co factor method to evaluate the inverse of the matrix B = $\begin{pmatrix} 1 & 3 & -5 \\ 3 & -1 & 2 \\ 1 & -2 & 1 \end{pmatrix}$ (6 marks)

c) For which values of k will the vector V=(1,2,k) be a member of subspace W where W = span{(3,0,-2) (2,-1,-5)} (4 marks)

QUESTION FOUR

a) Prove that a necessary and sufficient condition for two non-zero vectors $\underset{\smile}{x}$ and $\underset{\smile}{y}$ to be perpendicular is that their vector product should vanish (4 marks)

b) Find the area of triangle whose vertices are A(-1,8,0), B(7,-2,3) and C(-8, -6,0) (5 marks)

c) Find the equation of the plane through the points P(-4,-1,-1) Q(-2,0,1) and R(-1,-2,-3) (6 marks)

d) Given that $\underset{\smile}{a} = i + j + 2k$ b=2i + j +3k and c = 2i – 3j +4k. show that (axb) x c = b(a.c)-a(b.c)

(5 marks)

QUESTION FIVE

Consider the following basis of $\mathbb{R}^3$ $e = \{e_i = (1,0,0)\ e_2 = (0,1,0)\ e_3 = (0,0,1)\}$ and

$F = \{F_1 = (1,-1,0)\ \ F_2 = (1,0,-1)\ F_3 = (0,0,1)\}$

a) For every V=(a,b,c)∈ $\mathbb{R}^3$, Find $[V]_e$ and [V]F (4 marks)

b) Find transition matrix A from $\{e_i\}$ to $\{F_i\}$ and B from$\{F_i\}$ to $\{e_i\}$ (4 marks)

c) Verify that B=A^{-1} (3 marks)

d) Show that A-1[V]e=[V]F for any vector V∈ $\mathbb{R}^3$ (2 marks)

e) Show that the matrix representation $[T]_F = A^{-1}[T]_e A$ where T is defined by T(x,y,z)=(-2x+y+z, x-2y+z, x+y-2z) (7 marks)