MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCE

DEPARTMENT OF APPLIED SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

FOURTH YEAR **FIRST** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA2420 – DIFFERENTIAL GEOMETRY

DURATION: 2 HOURS

DATE: 11TH DECEMBER 2017

TIME: 9.00AM – 11.00AM

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A (Compulsory)

QUESTION ONE (30 Marks)

a) i) Find to the nearest degree the angle between $\underline{a} = 4\underline{i} - 2\underline{j} + 4\underline{k}$ and $\underline{b} = 3\underline{i} - 6\underline{j} - 2\underline{k}$

(3 Marks)

ii) For what values of 'a' are $\underline{P} = \underline{ai} - \underline{2j} + \underline{k}$ and $\underline{q} = 2a\underline{i} + \underline{aj} - 4\underline{k}$ perpendicular?

(2 Marks)

b) Find the vector product $\underline{A} \times (\underline{B} \times \underline{C})$given that $A = \underline{i} - \underline{2j} - 3\underline{k}, \underline{B} = \underline{2i} + \underline{j} - \underline{k}$ and $\underline{C} = \underline{i} + 3\underline{j} - 2\underline{k}$. (5 Marks)

c) Find an equation for the plane determined by the points $Q_1(2,-1,2)$ $Q_2(3,1,1)$ and $Q_3(-1,2,3)$ (5 Marks)

d) Determine the domain of the vector function $\underline{r}(t) = (cost, \ln(4-t), \sqrt{t+1})$ (2 Marks)

e) Show that the vector function $\underline{r}(t) = sint\underline{i} + cost\underline{j} + \sqrt{3}\underline{k}$ has a constant length and is orthogonal to its derivative. (4 Marks)

f) Find the tangent vector to the curve described by the equation $x = \cos t, y = \sin t, z = t$ at the point $(0,1,{}^{\pi}/_{2})$ (4 Marks)

g) Show that the curvature of a circle of radius 'r' is ${}^{1}/_{r}$. (5 Marks)

SECTION B (Answer any two questions)

QUESTION TWO (20 Marks)

a) Given that $\underline{y}(t) = a\cos t\underline{i} + a\sin t\underline{j} + bt\underline{k}$, find;

i. $\frac{d\underset{\sim}{y}}{dt}$ (1 Mark)

ii. $\left|\frac{d\underset{\sim}{y}}{dt}\right|$ (2 Marks)

iii. $\frac{d^2\underset{\sim}{y}}{dt^2}$ (1 Mark)

iv. $\left|\frac{d^2\underset{\sim}{y}}{dt^2}\right|$ (2 Marks)

b) Given that the equation in 2(a) above represents a space curve, express the equation in terms of the arc length parameter 's'. (5 Marks)

c) Given that $\underline{P} = (1+t^2)\underline{i} + \cos t\underline{j}\ and\ \underline{q} = \sin t\underline{i} + e^t\underline{k}$, find;

i. $\frac{d}{dt}(\underline{p} \cdot \underline{q})$ (2 Marks)

ii. $\frac{d}{dt}(\underline{p} \times \underline{q})$ (3 Marks)

d) Show that $\underline{r} = \underline{u} \cos kt + \underline{v} \sin kt, \underline{u}, \underline{v}$ being constant vectors is a solution to $\frac{d^2\underline{r}}{dt^2} = -k^2\underline{r}$

(4 Marks)

QUESTION THREE (20 Marks)

a) Find the curvature for the curve

$\underline{r}(t) = c \sin t\underline{i} + c \cos t\underline{j} + dt\underline{k}, c, d \geq 0, c^2 + d^2 \neq 0$ (9 Marks)

b) Find the normal vector $\underline{N}$ for the curve in Q3 (a) above. (6 Marks)

c) Find the curvature vector $\underline{K}$ and curvature $|K|$ on the curve $x = t\underline{i} + \frac{1}{2}t^2\underline{j} + \frac{1}{3}t^3\underline{k}$ at the point $t = 1$. (5 Marks)

QUESTION FOUR (20 Marks)

a) Find the unit tangent vector of the curve $\underline{r}(t) = (t^2 + 1)i + (4t - 3)j + (2t^2 - 6t)\underline{k}$ at the point where $t = 2$. (4 Marks)

b) A particle moves along the helix

$\underline{r}(t) = a \cos t\underline{i} + a \sin t\underline{j} + bt\underline{k}, a, b \geq 0, a^2 + b^2 \neq 0$

i. Find the velocity and acceleration of the particle. (2 Marks)

ii. Using the result of b(i) above, calculate the curvature and torsion of the helix

(10 Marks)

c) Compute the length of the arc $x = 2 \cosh 2t\underline{i} + 2 \sinh 2t\underline{j} + 4t\underline{k}$ between

$t = 0 \; and \; t = \pi$ (4 Marks)

QUESTION FIVE (20 Marks)

a) Find the equation of the tangent plane the normal line to the surface represented by $\underline{y} = u\underline{i} + v\underline{j} + (u^2 - v^2)\underline{k}$ at the point corresponding to $u = 1, v = 1$. (7 Marks)

b) Obtain the first fundamental forms on the surface represented by

$\underline{r} = (u + v)\underline{i} + (u - v)\underline{j} + uv\underline{k}$ (6 Marks)

c) Consider the surface represented by the equation of part 5(a) above. Obtain the second fundamental form on this surface. (7 Marks)