MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF APPLIED SCIENCES

UNIVERSITY ORDINARY EXAMINATION

2017/2018 ACADEMIC YEAR

FOURTH YEAR **FIRST** SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2401 – TOPOLOGY I

DURATION: 2 HOURS

DATE: 6TH DECEMBER, 2017

TIME: 9.00 – 11.00 A.M.

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION ONE - COMPULSORY

QUESTION ONE

(a) Define the following terms

i) Topology

ii) Accumulation points

iii) Closure of a set (6 marks)

(b) Consider the topology

$\tau = \{x, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ on $x = \{a, b, c, d, e\}$ and the subset

$A = \{b, c, d\}$ of x. Find the interior points of A. (3 marks)

(c) Show that the intersection $\tau_1 n \tau_2$ of any two topologies τ_1 and τ_2 on *x* is also a topology on x (4 marks)

(d) The class $\tau = \{x, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ define a topology on

$X = \{a, b, c, d, e\}$ consider the subset $A = \{a, b, c\}$ of x. Find the accumulation points of A.

(e) Consider the following topology on x

$x = \{a, b, c, d, e\}$

$\tau = \{x, \emptyset, \{a\}, \{a, 3, \}, \{a, b, c, d\}, \{a, c, d\}, \{a, b, e\}\}$

i) Find the interior points of the subset $A = \{a, b, c\}$ of x. (3 marks)

ii) Find the exterior points of A (2 marks)

iii) Find the boundary points of A (2 marks)

(f) Show that every point P in a discrete space x has a finite base (3 marks)

(g) Let x and y be topological spaces. Show that a function $f: x \rightarrow y$ is continuous if and only if it is continuous at every point $p \in x$ (4 marks)

SECTION TWO – ANSWER ANY TWO

QUESTION TWO

(a) List all topologies on x={a,b,c} which consist exactly four members (7 marks)

(b) Prove that the intersection $N \cap M$ of any two neighborhoods N and M of a point P is also a neighborhood of P (4 marks)

(c) Consider the following topology on $x = \{1,2,3,4,5\}$

$\tau = \{x, \emptyset\, \{1\}, \{1,2\}, \{1,2,4\}, \{1,2,3,4\}, \{1,2,5\}\}$ list the members of the relative Topology τ_A on $A = \{1,3,5\}$ (4 marks)

(d) List all possible topologies on the set x={a,b} (3 marks)

(e) Let x be an infinite set and let τ consist of $\emptyset$ and all subsets of x whose complements are countable if x is countable describe the topology determined by τ (2 marks)

QUESTION THREE

(a) Let x and y be topological spaces prove that a function $f: x \to y$ is continuous if and only if it is continuous at every point $p \in x$ (6 marks)

(b) Show that the property of being a Hoursdorff space is hereditary, that is every subspace of a Hoursdorff space is also Hoursdorff (7 marks)

(c) Prove that a set G is open if and only if it is a neighborhood of each of its points (7 marks)

QUESTION FOUR

(a) Prove that $\overline{A}$ = Int(A) ∪ b(A) (9 marks)

(b) Let x be a topological space. Show that the following conditions are equivalent

i) X is normal

ii) If H is an open superset of a closed set F, then there exist an open set G such that $F \subset G \subset G \subset H$ (11 marks)

QUESTION FIVE

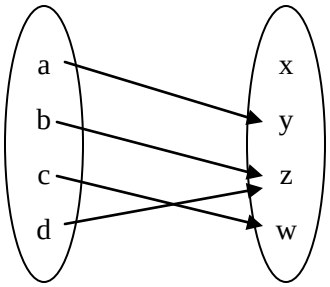
(a) Define the following terms as used in topology

i) Basis of a topology

ii) Relative topology

iii) Continuous function (6 marks)

(b) Consider the following topologies on $x = \{a, b, c, d\}$ and $y = \{x, y, z, w\}$ $\tau = \{x, \emptyset\, \{a\}, \{a, b\}, \{a, b, c\}\}$ and $T = \{y, \emptyset\, \{y\}, \{x, y\}, \{y, z, w\}\}$ and the function $f: x \to y$ defined by f(a)=y, f(b)=z, f(c)=w and f(d)=z. Determine whether f is continuous or not (4 marks)



Determine whether f is continuous or not (4 marks)

(c) Let x be an indiscrete space

i) Determine the closed subsets of x (2 marks)

ii) Determine this closure of any subset A of x (3 marks)

iii) Determine the dense subsets of x (2 marks)

(d) Let the real function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Show that f is not open (3 marks)